Practice Test 3 on Section 4.1-4.6
Math 1392 - Trigonometry - Fall 2015

Instructions: Calculators are not allowed on this test. Each problem is worth 10 points.

1. Determine whether the following functions are odd, even or neither:

a. $f(x) = x\sin(x)$

b. $f(x) = \dfrac{\cos(x)}{x^2+1}$

c. $f(x) = \sin(x) + \tan(x)$

2. Factor the following trigonometric expressions:

a. $\sin^2(x) + 4\sin(x) - 5$

b. $4\sin(x)\cos^2(x) - \sin(x)$

3. Verify the following identity:

$$\frac{\csc(x)}{\sec(x)} - \frac{\sec(x)}{\csc(-x)} = \sec(x)\csc(x)$$

4. Verify the following identity:

$$\sec(x) - \tan(x)\sin(x) = \cos(x)$$

5. Verify the following identity:

$$\frac{\tan(x)-1}{\tan(x)+1} = \frac{\sin(x)-\cos(-x)}{\sin(x)+\cos(-x)}$$

6. Verify the following identity:

$$\frac{\cos(\alpha+\beta)}{\sin(\alpha-\beta)} = \frac{1-\tan\alpha\tan\beta}{\tan\alpha-\tan\beta}$$

7. Determine the exact value of $\sin(105^o)$.

8. Determine the exact value of $\cos(5\pi/12)$

9. Simplify the following trigonometric expression completely:

$$\cos(\alpha+\beta) - \cos(\alpha-\beta)$$

10. Use the co-function identities to complete the following matching

_____ 1) $\cos(16^o)$	A. $\sin(20^o)$
_____ 2) $\cos(70^o)$	B. $\sin(74^o)$
_____ 3) $\cos(\pi/10)$	C. $\sin(2\pi/5)$
_____ 4) $\cos(\pi/5)$	D. $\sin(3\pi/10)$

Basic Identities

$$\tan(x) = \frac{\sin(x)}{\cos(x)} \qquad \cot(x) = \frac{\cos(x)}{\sin(x)}$$

Reciprocal Identities

$$\sec(x) = \frac{1}{\cos(x)} \qquad \csc(x) = \frac{1}{\sin(x)} \qquad \cot(x) = \frac{1}{\tan(x)}$$

Pythagorean Identities

$$\sin^2(x) + \cos^2(x) = 1 \qquad \tan^2(x) + 1 = \sec^2(x) \qquad 1 + \cot^2(x) = \csc^2(x)$$

Odd/Even Identities

$$\sin(-x) = -\sin(x) \qquad \tan(-x) = -\tan(x) \qquad \sec(-x) = \sec(x)$$
$$\cos(-x) = \cos(x) \qquad \csc(-x) = -\csc(x) \qquad \cot(-x) = -\cot(x)$$

Sum and Difference Identities

$$\cos(\alpha+\beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta)$$
$$\cos(\alpha-\beta) = \cos(\alpha)\cos(\beta) + \sin(\alpha)\sin(\beta)$$
$$\sin(\alpha+\beta) = \sin(\alpha)\cos(\beta) + \cos(\alpha)\sin(\beta)$$
$$\sin(\alpha-\beta) = \sin(\alpha)\cos(\beta) - \cos(\alpha)\sin(\beta)$$

$$\tan(\alpha+\beta) = \frac{\tan(\alpha)+\tan(\beta)}{1-\tan(\alpha)\tan(\beta)}$$
$$\tan(\alpha-\beta) = \frac{\tan(\alpha)-\tan(\beta)}{1+\tan(\alpha)\tan(\beta)}$$

CoFunction Identities

$$\cos(\pi/2-u) = \sin(u) \qquad \tan(\pi/2-u) = \cot(u) \qquad \csc(\pi/2-u) = \sec(u)$$
$$\sin(\pi/2-u) = \cos(u) \qquad \sec(\pi/2-u) = \csc(u) \qquad \cot(\pi/2-u) = \tan(u)$$

Answers to Practice Test

1.a. $f(x) = x \cdot \sin x$

$f(-x) = (-x)\sin(-x)$

$= -x(-\sin x)$

$= x \cdot \sin x = f(x)$

even

1.b. $f(x) = \dfrac{\cos x}{x^2+1}$

$f(-x) = \dfrac{\cos(-x)}{(-x)^2+1}$

$= \dfrac{\cos x}{x^2+1} = f(x)$

even

1c. $f(x) = \sin x + \tan x$

$f(-x) = \sin(-x) + \tan(-x)$

$= -\sin x - \tan x$

$= -(\sin x + \tan x) = -f(x)$

odd

2.a. $\sin^2(x) + 4\sin x - 5$

$(\sin x + 5)(\sin x - 1)$

2b. $4\sin x \cos^2 x - \sin x$

$\sin x(4\cos^2 x - 1) = \sin x(2\cos x - 1)(2\cos x + 1)$

3. $\dfrac{\csc x}{\sec x} - \dfrac{\sec x}{\csc(-x)} = \sec x \cdot \csc x$

$\dfrac{\frac{\cos x}{1}\,\frac{1}{\sin x}}{\frac{\cos x}{1}\,\frac{1}{\cos x}} + \dfrac{\frac{1}{\cos x}}{\frac{+1}{\sin x}}\,\dfrac{\frac{\sin x}{1}}{1} = \sec x \cdot \csc x$

$\left(\dfrac{\cos x}{\cos x}\right)\dfrac{\cos x}{\sin x} + \dfrac{\sin x}{\cos x}\left(\dfrac{\sin x}{\sin x}\right) = \dfrac{1}{\cos x} \cdot \dfrac{1}{\sin x}$

$\dfrac{\cos^2 x + \sin^2 x}{\cos x \sin x \cancel{\cos x}} = \dfrac{1}{\cos x \cdot \sin x}$

$\dfrac{1}{\cos x \sin x} = \dfrac{1}{\cos x \cdot \sin x}$

4. $\sec x - \tan x \sin x = \cos x$

$$\frac{1}{\cos x} - \frac{\sin x}{\cos x} \cdot \frac{\sin x}{1} = \cos x$$

$$\frac{1-\sin^2 x}{\cos x} = \cos x$$

$$\frac{\cos^2 x}{\cos x} = \cos x$$

5. $$\frac{\tan x - 1}{\tan x + 1} = \frac{\sin x - \cos(-x)}{\sin x + \cos(-x)}$$

$$\frac{\frac{\sin x}{\cos x} - \frac{1}{1}\left(\frac{\cos x}{\cos x}\right)}{\frac{\sin x}{\cos x} + \frac{1}{1}\left(\frac{\cos x}{\cos x}\right)} = \frac{\sin x - \cos x}{\sin x + \cos x}$$

$$\frac{\frac{\sin x - \cos x}{\cancel{\cos x}}}{\frac{\cancel{\sin x + \cos x}}{\cancel{\cos x}}} \cdot \frac{\frac{\cancel{\cos x}}{\cancel{\sin x + \cos x}}}{\frac{\cancel{\cos x}}{\cancel{\sin x + \cos x}}} = \frac{\sin x - \cos x}{\sin x + \cos x}$$

6. $$\frac{\cos(\alpha+\beta)}{\sin(\alpha-\beta)} = \frac{1 - \tan\alpha \tan\beta}{\tan\alpha - \tan\beta}$$

$$\frac{\cos\alpha\cos\beta - \sin\alpha\sin\beta}{\sin\alpha\cos\beta - \cos\alpha\sin\beta} = \frac{1 - \frac{\sin\alpha}{\cos\alpha}\frac{\sin\beta}{\cos\beta}}{\frac{\sin\alpha}{\cos\alpha} - \frac{\sin\beta}{\cos\beta}}$$

$$= \frac{\frac{\cos\alpha\cos\beta - \sin\alpha\sin\beta}{\cancel{\cos\alpha\cos\beta}}}{\frac{\cancel{\sin\alpha\cos\beta - \cos\alpha\sin\beta}}{\cancel{\cos\alpha\cos\beta}}} \cdot \frac{\frac{\cancel{\cos\alpha\cos\beta}}{\sin\alpha\cos\beta - \cos\alpha\sin\beta}}{\frac{\cancel{\cos\alpha\cos\beta}}{\cancel{\sin\alpha\cos\beta - \cos\alpha\sin\beta}}}$$

7. $\sin 105^\circ = \sin(60^\circ + 45^\circ) = \sin 60^\circ \cos 45^\circ + \cos 60^\circ \sin 45^\circ$

$$= \frac{\sqrt{3}}{2}\,\frac{\sqrt{2}}{2} + \frac{1}{2}\,\frac{\sqrt{2}}{2} = \frac{\sqrt{6}+\sqrt{2}}{4}$$

8. $\cos\frac{5\pi}{12} = \cos\left(\frac{3\pi}{12} + \frac{2\pi}{12}\right) = \cos\left(\frac{\pi}{4} + \frac{\pi}{6}\right) = \cos\frac{\pi}{4}\cos\frac{\pi}{6} - \sin\frac{\pi}{4}\sin\frac{\pi}{6}$

$$= \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2}\cdot\frac{1}{2} = \frac{\sqrt{6}-\sqrt{2}}{4}$$

9. $\cos(\alpha+\beta) - \cos(\alpha-\beta)$

$(\cos\alpha\cos\beta - \sin\alpha\sin\beta) - (\cos\alpha\cos\beta + \sin\alpha\sin\beta)$

$\cos\alpha\cos\beta - \sin\alpha\sin\beta - \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$-2\sin\alpha\sin\beta$

10. 1) $\cos 16^\circ = \sin(90^\circ - 16^\circ) = \sin 74^\circ$ B.

2) $\cos 70^\circ = \sin(90^\circ - 70^\circ) = \sin 20^\circ$ A.

3) $\cos\frac{\pi}{10} = \sin\left(\frac{\pi}{2} - \frac{\pi}{10}\right) = \sin\left(\frac{5\pi}{10} - \frac{\pi}{10}\right) = \sin\left(\frac{4\pi}{10}\right) = \sin\left(\frac{2\pi}{5}\right)$ C.

4) $\cos\left(\frac{\pi}{5}\right) = \sin\left(\frac{\pi}{2} - \frac{\pi}{5}\right) = \sin\left(\frac{5\pi}{10} - \frac{2\pi}{10}\right) = \sin\left(\frac{3\pi}{10}\right) =$ D.