MATH 4305 - Applied Mathematics I
Homework 2 - Method of Undetermined Coefficients
Due - Monday, September 28, 2015

Identify any singular points in the following linear differential equations. Determine whether the singular point is a regular singular point.

1. $(x-2)y'' + 3(x^2 - 3x + 2)y' + (x-2)^2 y = 0$
2. $(x+1)y'' + \frac{1}{x}y' + xy = 0$
3. $x^3 y'' + xy = 0$
4. $(x+1)^3 y'' + (x^2-1)(x+1)y' + (x-1)y = 0$
5. $x^3 y'' + 2x^2 y' + y = 0$
6. $x^3 y'' + 5xy' + x(x-1)y = 0$

For problems 7-12, find the specified coefficients and the recurrence relationship. For problems 7-9, write the solution as

$$y(x) = a_0 + a_1(x-x_0)^2 + a_2(x-x_0)^2 + a_3(x-x_0)^3 + a_4(x-x_0)^4 + \cdots$$

with the coefficients that you have determined. For problems 8-10, write the solution in factored form, showing the two linearly independent solutions, as

$$y(x) = a_0 y_1(x) + a_1 y_2(x) + y_p(x)$$

where $y_1(x)$, $y_2(x)$ and $y_p(x)$ are power series functions.

Use the Method of Undetermined Coefficients to determine the solution to the following initial value problems:

7. $y' + x^2 y = 0$ with $y(0) = 2$. Find coefficients a_0 through a_6.

8. $y'' - 2xy' + 3y = 0$ with $y(0) = 1$ and $y'(0) = 2$. Find coefficients a_0 through a_4.

9. $y'' + (x+1)y = 0$ with $y(0) = 1$ and $y'(0) = 2$. Find coefficients a_0 through a_5.

Use the Method of Undetermined Coefficients to determine the general solution to the following differential equations. Factor the answers into distinct solutions involving the initial coefficients a_0, a_1 and a_2, if necessary, along with the particular solution, if the equation is non-homogeneous.

10. $y'' + xy' + y = 5$ centered at $x_0 = 0$. Find coefficients a_0 through a_5.

11. $y'' + x^2 y' + xy = x$ centered at $x_0 = 0$. Find coefficients a_0 through a_7.

12. $y''' - xy' + 2y = 0$ centered at $x_0 = 0$. Find coefficients a_0 through a_7.