Solve each differential equation using Laplace Transforms. Your answers should include the following:

1. Laplace transform of the differential equation;

2. The expression for $Y(s)$ before and after finding the partial fraction decomposition;

3. The solution to the differential equation.

For each problem, assume $y(0) = a_0$, $y'(0) = a_1$ and $y''(0) = a_2$, if necessary.

1. $y' + 3y = 0$

2. $y' + 3y = 2$

3. $y' + 3y = e^{-3x}$

4. $y' + 3y = \cos(3x)$

5. $y'' + 2y' - 3y = \sin(2x)$

6. $y'' + 2y' + 5y = 3e^{-2x}$

7. $y'' - 6y' = (x - 4)$

8. $y''' - y = 5$

9. $y'(x) + 2y(x) = g(x)$ where $g(x) = \begin{cases} 0 & x < 1 \\ x^2 & x > 1 \end{cases}$

10. $y'(x) + 3y(x) = g(x)$ where $g(x) = \begin{cases} 0 & x < 2 \\ x^2 - 2x & x > 2 \end{cases}$

11. $y'(x) + 5y(x) = 2\delta(x - 3)$

12. $y''(x) - 3y'(x) - 4y(x) = \delta(x - 5)$