MATH 4305 - Applied Mathematics I
Project 2 - Power Series Solution of Classic Differential Equations
Fall 2015 - Dr. Clarence Burg

Introduction

Several differential equations have special significance because of their relationship to other branches of mathematics and the other sciences. In this project, you will investigate some of the following differential equations:

1. Hermite Equation: $y'' - 2xy' + 2Ny = 0$
2. Chebyshev's Equation: $(1 - x^2)y'' - xy' + N^2 y = 0$
3. Legendre's Equation: $(1 - x^2)y'' - 2xy' + N(N + 1)y = 0$
4. Laguerre's Equation: $xy'' + (1 - x)y' + Ny = 0$
5. Bessel's Equation: $x^2 y'' + xy' + (x^2 - N^2)y = 0$

These equations involve a constant positive integer N, which creates a family of equations for each of these classic differential equations. For each positive value N, there is a Nth degree polynomial which satisfies the differential equation. There may be another, unrelated infinite series solution to the differential equation, but this solution is often ignored. Also, there is a scaling factor for the solution to the differential equation, in order to satisfy some other criterion, such as some normality condition (i.e., the length of the polynomial is 1 when integrated over a certain interval in a certain way).

We completed the analysis of the Hermite differential equation in class. In this project, you will study three of the other four differential equations. You may select either Chebyshev's equation or Legendre's equation, you must analyze Laguerre's equation and Bessel's equation. (Please note that Bessel functions satisfy Bessel's equation and that N may be other values than integers for this equation. We will only be studying positive integer values for N.)

For Chebyshev's equation, Legendre's equation and Laguerre's equation, please complete the following:

1. Investigate the significance of each equation and the polynomial solutions associated with each equation. Include the main applications for each equation.
2. Determine the initial terms for power series solution, centered at $x = 0$, along with the recurrence relationship.
3. Determine the Nth degree polynomial solution for $N = 0$ through $N = 6$.
4. Determine the scaling factor that scales these polynomial solutions into the common form for each polynomial.

For Bessel's equation, please complete the first two steps listed above. Its power series does not terminate, so find the first four non-zero terms in the solution to Bessel's equation. (To be consistent with online definitions, $a_0 = 1/N!$ for Bessel's equation.)

The report for Project is due on Friday, October 9, 2015.