Math 4340 – Numerical Methods

Homework 1.4

Due Tuesday, September 15, 2015

1. Show that the appropriate Richardson extrapolation formula will improve the accuracy of numerical results given by , if the numerical method is third-order accurate and  and  have been calculated. In other words, assume that  is third-order accurate, plug into the appropriate Richardson extrapolation formula and show that the leading order error term cancels out.

2. Determine the order of accuracy of the numerical scheme used to approximate the derivative of  at x=1, if the results are

|  |  |
| --- | --- |
| Discretization Size | Approximation to the derivative |
| h=0.4 | 1.36892517381 |
| h=0.2 | 1.36144793680 |
| h=0.1 | 1.35970978684 |
| h=0.05 | 1.35928265093 |
| h=0.025 | 1.35917631848 |
| h=0.0125 | 1.35914976343 |

Use the formula for Richardson extrapolation, appropriate for this order of accuracy, to improve the overall accuracy of this approximation. What is the order of accuracy for this new approximation? Does the result surprise you?

3. Calculate the order of accuracy of the numerical scheme used to generate the following data

|  |  |
| --- | --- |
| Discretization Size | Approximation to the derivative |
| h=0.2 | 0.452304118452 |
| h=0.1 | 0.453769702729 |
| h=0.05 | 0.453969994909 |
| h=0.025 | 0.453996171790 |
| h=0.0125 | 0.453999516600 |

Use the formula for Richardson extrapolation, appropriate for this order of accuracy, to improve the overall accuracy of this approximation. What is the order of accuracy for this new approximation? Does the result surprise you?

4. Calculate the order of accuracy of the numerical scheme used to generate the following data:

|  |  |
| --- | --- |
| Discretization Size | Approximation to the derivative |
| 2h | 1.076720909 |
|  | 1.061734001 |
| h | 1.054361018 |

Make sure that you use the appropriate ratio for this problem.