MATH 4340 - Numerical Methods
Homework 2.4 - Newton's Method
Due - Thursday, November 19, 2015

Instructions: For each of these problems, please tabulate your answers using the following format:

k	p_k	$e_k = \lvert p_k - P \rvert$	e_{k+1}/e_k^R
0			
1			
2			
3			
4			

Record all answers to at least 10 significant digits, except for the ratio of errors.

1. Perform 4 iterations of Newton's Method starting at $p_0 = 3.0$ to identify the root of $f(x) = x^2 + 5x - 14$. Determine the error between the iteration p_k and the root at $x = 2$, and determine the ratio of these errors that shows that this process converges quadratically to 2.

2. Perform 6 iterations of Newton's Method starting at $p_0 = 3.0$ to identify the root of $f(x) = x^3 - 6x^2 + 12x - 8$. Determine the error between the iteration p_k and the root at $x = 2$, and determine the ratio of these errors that shows that this process converges linearly to 2.

3. Perform 6 iterations of Newton's Method starting at $p_0 = 2.5$ to identify the root of $f(x) = x^5 - 7x^3 + 6x^2$. Determine the error between the iteration p_k and the root that is identified by this process (it should be an integer). Based on its convergence behavior, determine whether this iteration converges linearly or quadratically to the root, and verify this claim by determining the ratio of error consistent with this observation and showing that this ratio converges to a non-zero constant value.

4. Perform 6 iterations of Newton's Method starting at $p_0 = 2.5$ to identify the root of $f(x) = x^4 - x^3 - 8x^2 + 12x$. Determine the error between the iteration p_k and the root that is identified by this process (it should be an integer). Based on its convergence behavior, determine whether this iteration converges linearly or quadratically to the root, and verify this claim by determining the ratio of error consistent with this observation and showing that this ratio converges to a non-zero constant value.

5. By using a function that only involves addition, subtraction and multiplication, use Newton's method to estimate the value of $\sqrt{5}$ when starting at $p_0 = 2$.

6. By using a function that only involves addition, subtraction and multiplication, use Newton's method to estimate the value of $\sqrt[3]{6}$ when starting at $p_0 = 2$.