MATH 4340 - Numerical Methods
Homework 7.1 - Quadrature Rules
Due - Tuesday, October 6, 2015

Use these two integrals for each of the problems given below

$$\int_0^1 (1 + e^{-x} \cos(4x))\, dx \qquad\qquad \int_0^1 \sin(\sqrt{x})\, dx$$

(Note: Problems 1-5 are reworded problems from the textbook by Mathews and Fink.)

1. Use the trapezoid rule with $h = 1$ (or 1 interval) to estimate the value of each of the integrals.

2. Use Simpson's rule with $h = 1/2$ (or 2 intervals) to estimate the value of each of the integrals.

3. Use Simpson's 3/8's rule with $h = 1/3$ (or 3 intervals) to estimate the value of each of the integrals.

4. Use Boole's rule with $h = 1/4$ (or 4 intervals) to estimate the value of each of the integrals.

5. Use the trapezoid rule, Simpson's rule and Boole's rule to estimate the value of each of the integrals using $h = 1/4$. Note that you have already performed the Boole's rule calculation in problem 4. To perform the trapezoid rule calculation, you will need to apply the trapezoid rule 4 times, once to each of the following intervals $[0, 1/4]$, $[1/4, 1/2]$, $[1/2, 3/4]$ and $[3/4, 1]$. Similarly, for the Simpson's rule, you will need to apply it to the intervals $[0, 1/2]$ and $[1/2, 1]$, in order to use $h = 1/4$.

6. Determine the degree of precision for the midpoint rule which is given below:

$$\int_{x_0}^{x_0+h} f(x)\, dx \approx h f(x_0 + h/2)$$

And determine the error term associated with this integration formula.

7. Determine the degree of precision for the following rule which is called the two-point Gauss quadrature rule:

$$\int_{-1}^{1} f(x)\, dx \approx \left(f(-1/\sqrt{3}) + f(1/\sqrt{3}) \right)$$

Its precision is much higher than the trapezoid rule.