MATH 4340 - Numerical Methods
Homework 7.2 - Composite Quadrature Rules
Due - Friday, April 10, 2015

Please note that all of these problems are reworded from Section 7.2 of our textbook by Mathews and Fink.

1. Use 8 intervals to estimate the value of the following integral using the composite trapezoid rule, Simpson's rule and Boole's rule:

$$\int_0^4 x^2 e^{-x}\, dx$$

2. The arclength of the curve $y = f(x)$ over the interval $a \leq x \leq b$ is given by the following integral:

$$\int_a^b \sqrt{1 + (f'(x))^2}\, dx$$

Use 8 intervals to estimate the arc length of the function $y = \sin(x)$ over the interval $0 \leq x \leq \pi/4$ using the composite trapezoid rule, Simpson's rule and Boole's rule.

3. Determine the number of intervals N and the step size h so that the result from the trapezoid rule is guaranteed to have an error of less than 10^{-7} for the following intervals

$$\int_{-\pi/6}^{\pi/6} \cos(x)\, dx$$
$$\int_2^3 \frac{1}{5-x}\, dx$$
$$\int_0^2 xe^{-x}\, dx$$

4. Determine the number of intervals N and the step size h so that the result from Simpson's rule is guaranteed to have an error of less than 10^{-7} for the following intervals

$$\int_{-\pi/6}^{\pi/6} \cos(x)\, dx$$
$$\int_2^3 \frac{1}{5-x}\, dx$$
$$\int_0^2 xe^{-x}\, dx$$

5. Determine the number of intervals N and the step size h so that the result from Boole's rule is guaranteed to have an error of less than 10^{-7} for the following intervals

$$\int_{-\pi/6}^{\pi/6} \cos(x)\, dx$$

$$\int_{2}^{3} \frac{1}{5-x}\, dx$$

$$\int_{0}^{2} xe^{-x}\, dx$$

6. The results of using the trapezoid rule to estimate the value of the definite integral $\int_{-0.1}^{0.1} \cos(x)\, dx = 2\sin(0.1) \approx 0.1996668333$ are given in the following table:

N	h	$N(h)$	$E(h)$	p
1	0.2	0.1990008		
2	0.1	0.1995004		
4	0.05	0.1996252		
8	0.025	0.1996564		
16	0.0125	0.1996642		

Determine the absolute error with each calculation and the observed order of accuracy of the numerical calculation. Confirm that the composite trapezoid rule is second order accurate for this integral.

7. The results of using Simpson's rule to estimate the value of the definite integral $\int_{-0.75}^{0.75} \cos(x)\, dx = 2\sin(0.75) \approx 1.363277520$ are given in the following table:

N	h	$N(h)$	$E(h)$	p
2	0.75	1.3658444		
4	0.375	1.3634298		
8	0.1875	1.3632869		
16	0.09375	1.3632781		

Determine the absolute error with each calculation and the observed order of accuracy of the numerical calculation. Confirm that the composite Simpson's rule is fourth order accurate for this integral.