MATH 4340 - Numerical Methods
Project 1 - Numerical Differentiation
Fall 2015 - Dr. Clarence Burg

In this report, you will study the error and the accuracy of several different numerical methods for approximating the first and second derivatives. You will use the formula for the observed order of accuracy

$$p = \frac{\ln\left(\frac{N(r^2h)-N(rh)}{N(rh)-N(h)}\right)}{\ln(r)}$$

to determine the observed order of accuracy of these methods. Once the implementation has been verified by showing that the observed order of accuracy agrees with the theoretical order of accuracy, you can use Richardson extrapolation on the numerical results to increase the order of accuracy. The Richardson extrapolation formula using a refinement ratio of r on a numerical method of order p is

$$N_{RE}(h) = \frac{r^p N(h) - N(rh)}{r^p - 1}$$

Project Report

Your project report should include the following 5 sections.

<u>Section 1: Applications.</u>

Obviously, the calculation of the derivative of a function is one of the basic concepts of Calculus, and there are rules for calculating the derivative analytically for a huge number of functions. So, why do we need to study the numerical differentiation of a function? What are the applications of numerical differentiation? In fact, the calculation of derivatives numerically is one of the key concepts of numerical methods. So, explain why it is important by discussing its numerous applications.

<u>Section 2: Description of the mathematical concepts and algorithms.</u>

You need to include the numerical approximations used within this report, along with the leading order error term for each of these approximations. These formula include the following approximations to the first derivative:

1. Forward difference formula
2. Double forward difference formula
3. Central difference formula
4. Fourth order central difference formula

Please also include the formula for the second derivative, include the second order central difference formula, the forward difference formula and the fourth order central difference formula for the second derivative.

Section 3: Translation of mathematical algorithm into pseudo-code

Since these formula are so simple, there is no need to write pseudo-code for them.

Section 4: Verification Results

This section will focus on the observed order of accuracy of these techniques, which should agree with the theoretical order of accuracy. You will also use Richardson Extrapolation to improve the numerical results. If you are using R, C/C++, Java or some other programming language, you need to report all 16 digits of your numerical result.

For the function $f(x) = \dfrac{\sin(x)}{1+e^x}$, determine the exact value of the **first derivative** at $x = 0.4$. Then, estimate the derivative of this function at $x = 0.4$ using step sizes ranging from $h = 0.1$ down to $h = 0.003125$ and complete the following table.

h	$N(h)$	$E(h)$	p	$N_{RE}(h)$	p_{RE}
0.1			NA	NA	NA
0.05			NA		NA
0.025					NA
0.0125					
0.00625					
0.003125					

Create four separate tables, using the forward difference, the second order double forward difference, second order central difference and fourth order central difference formula. Determine the observed order of accuracy and state the integer value of the order of accuracy. Use the appropriate Richardson extrapolation formula to generate more accurate results for each approximation and calculate the observed order of accuracy for the Richardson extrapolation result. Do you observe any relationship between the results of the two different forward difference formula, especially in relationship to the use of Richardson extrapolation? For the two different central difference formula? If so, what is the relationship?

For the function $f(x) = \dfrac{\sqrt{4+\sin(x)}}{2+\cos(x)}$, determine the exact value of the **second derivative** at $x = 1.0$. Then, estimate the second derivative of this function at $x = 1.0$ using step sizes ranging from $h = 0.1$ down to $h = 0.003125$ and complete the following table.

h	$N(h)$	$E(h)$	p	$N_{RE}(h)$	p_{RE}
0.1			NA	NA	NA
0.05			NA		NA
0.025					NA
0.0125					
0.00625					
0.003125					

Create three separate tables, using the forward difference, the second order central difference and fourth order central difference formula. Determine the observed order of accuracy and state the integer value of the order of accuracy. Use the appropriate Richardson extrapolation formula to generate more accurate results for each approximation and calculate the observed order of accuracy for the Richardson extrapolation result. Do you observe any relationship between the results of the two different central difference formula? If so, what is the relationship?

Section 5: Validation Results

There is no validation exercises for this project.

The Project Report for Project 1 is due on Tuesday, Sept 22, 2015.