A Numerical Design Method for Open-Channel Flow

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Abstract

With the growth of urban centers and the resulting changes in infrastructure, the capacity of many existing storm water management system has been stretched to its limit. Altered surface hydrology increases surface runoff which reaches drainage channels more quickly, resulting in greater system loads. To handle this increased flow, many of these channels need to be redesigned. Complicating this process are the restrictions on channel designs due to other structures, such as roads, bridges and buildings, because alterations to these structures can be expensive. Designing high-velocity channels to handle the increased water flow while minimizing the alterations to existing structures is a formidable task, with the current design technique being a trial and error approach.

In this paper, we present an efficient, deterministic design method, which applies the adjoint variable formulation of direct differentiation to a computational, open-channel flow model (HIVEL2D) in order to obtain the derivative of an objective function with respect to the design variables. From these derivatives, the design variables are modified with the goal of minimizing the objective function. The test cases involve channel contraction problems with one, two and three design variables.

Introduction

Various fluid dynamics codes simulate the flow of water through man-made open channels. Designers use these codes, in conjunction with scale models, to analyze the merits of a particular channel design. By using their expertise, they adjust the channel design in order to obtain the desired effects. Unfortunately, this trial and error process relies heavily on the designer, is not deterministic and provides no information about the influence of design parameters on the flow. Flows through

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high-velocity channels produce hydraulic phenomena, such as hydraulic jumps and standing waves, in conjunction with changes in the channel shape. By modifying the channel design, the undesirable effects of these hydraulic phenomena can be reduced or eliminated.

For a given problem, the channel design is determined parametrically by one or more design variables $\vec{\beta}$. An appropriate objective function F is chosen and is usually a function of the flow variables Q, which are dependent on the channel design. By adjusting the design variables, the value of the objective function can be minimized. The derivative of the objective function with respect to each of the design variables is approximated with the adjoint variable formulation of direct differentiation (Huddleston, Soni, 1994), and the design variables are adjusted accordingly, which modifies the channel design.

To simulate the flow through the channel, HIVEL2D(Stockstill, Berger, 1994) is used. This code solves the depth-averaged, two-dimensional, viscous shallow water equations, ignoring Coriolis or wind effects. The code uses a Newton-Raphson iteration method to advance the solution in time and a Petrov-Galerkin finite element formulation for the spatial domain.

A conceptually simple estimate for each derivative term $\frac{dF}{d\beta_i}$ can be made by its finite difference approximation, $\frac{dF(\vec{\beta})}{d\beta_i} \approx \frac{F(\vec{\beta}+\Delta\beta_i)-F(\vec{\beta})}{\beta_i}$. For a problem with n design variables, $n+1$ function evaluations would be needed. But each function evaluation requires a steady-state simulation. Rather, by using the adjoint variable formulation, only one steady-state simulation and one solution of a linear adjoint system are needed, which is computationally less expensive. Once the design space gradient is obtained, the method of steepest descent is employed to update the design variables.

This results presented in this paper demonstrate that the adjoint variable formulation can be applied to open-channel design optimization. Hence, gradient-based, channel design optimization can be accomplished with a much more reasonable amount of computation. This technique was verified using three design problems where the optimal solution was known and then applied to two channel contraction design problems where the optimal design was not known.

Sensitivity Equations

For open-channel flow optimization problems, the objective function F is directly a function of the flow variables Q. These flow variables depend on the channel design, which is determined from the design variables $\vec{\beta}$. Thus, $F[Q(\vec{\beta})]$ where Q denotes a vector of flow variables and $\vec{\beta}$ denotes a vector of design variables. The flow variables Q represent the steady-state values for a particular set of design variables $\vec{\beta}$. In order to minimize the objective function, the derivative of F with respect to each design variable β_i is needed, which is derived below.

After discretization of the flow domain, an implicit, steady-state algorithm yields a system of nonlinear algebraic equations of the form $[W(Q(\beta), X(\beta))] = [0]$ which must be solved to obtain the flow variables. The code HIVEL2D solves a similar time-dependent set of equations W^* by updating the flow variables, where $\Delta Q^n = Q^{n+1} - Q^n$, using

$$\left[\frac{\partial W^*(Q^n)}{\partial Q}\right]\left[\Delta Q^n\right] = -\left[W^*(Q^n)\right] \tag{1}$$

The derivative of F with respect to the design variable β_i is

$$\frac{dF}{d\beta_i} = \left[\frac{\partial F}{\partial Q}\right]^T \left[\frac{\partial Q}{\partial \beta_i}\right] \tag{2}$$

The converged, steady-state solution to the algebraic system of equations, which represents the finite element solution algorithm, yields

$$\left[\frac{dW}{d\beta_i}\right] = [0] = \left[\frac{\partial W}{\partial Q}\right]\left[\frac{\partial Q}{\partial \beta_i}\right] + \left[\frac{\partial W}{\partial X}\right]\left[\frac{\partial X}{\partial \beta_i}\right] \tag{3}$$

Multiplying eq. (3) by an arbitrary adjoint vector $[\lambda]$ and adding to eq. (2) yields

$$\frac{dF}{d\beta_i} = \left(\left[\frac{\partial F}{\partial Q}\right]^T + [\lambda]^T \left[\frac{\partial W}{\partial Q}\right]\right)\left[\frac{\partial Q}{\partial \beta_i}\right] + [\lambda]^T \left[\frac{\partial W}{\partial X}\right]\left[\frac{\partial X}{\partial \beta_i}\right] \tag{4}$$

By choosing $[\lambda]$ such that

$$\left[\frac{\partial F}{\partial Q}\right]^T + [\lambda]^T \left[\frac{\partial W}{\partial Q}\right] = [0]^T \tag{5}$$

eq. (4) simplifies to

$$\frac{dF}{d\beta_i} = [\lambda]^T \left[\frac{\partial W}{\partial X}\right]\left[\frac{\partial X}{\partial \beta_i}\right] = [\lambda]^T \left[\frac{dW}{d\beta_i}\right]\Bigg|_{Q fixed} \tag{6}$$

The vector $[\lambda]$ is calculated via the equation $\left[\frac{\partial W}{\partial Q}\right]^T [\lambda] = -\left[\frac{\partial F}{\partial Q}\right]^T$. Using eq. (1), HIVEL2D can be used to solve for $[\lambda]$ by changing the right-hand side and using the transpose of $\left[\frac{\partial W}{\partial Q}\right]$. Once $[\lambda]$ is determined, $\frac{dF}{d\beta_i}$ can be approximated using the difference quotient for $\frac{dW}{d\beta_i}$.

The primary advantage of the discrete adjoint variable formulation of direct differentiation is that for each iteration in the design optimization algorithm, the method of direct differentiation requires only one steady-state simulation via eq. (1) and one solution of eq. (5) for each objective function, regardless of the number of design variables.

Initial Verification
To verify that the derivatives produced by the aforementioned technique would lead the design process to the optimal solution, three simple test cases were used, where the optimal design was known. The first of these test cases involved a channel contraction whose wall curve was governed by a polynomial passing through three points, each point representing a design variable. The objective function was designed so that its only minimum occurred for the target set of (2.0, 5.0, 7.0). Starting at (1.0, 3.0, 8.0) and proceeding for 62 iterations, the design variables were (1.947, 4.995, 6.993), which closely approximated the target.

For the second and third test cases as well as the two design variable problem, the channel contracts from 40 ft to 30 ft. The inflow boundary conditions are set at a velocity of 28.375 ft/sec and a depth of 1.0 ft, resulting in a Froude number of 5.0. Since uniform flow is desired, the objective function was of the form

$$F(\beta) = \sum_{10 \text{ columns}} \left(\sum_{row=2}^{9} \left(depth(col, row) - depth(col, row-1) \right)^2 \right) \tag{7}$$

The objective function should be zero for uniform flow, but as purely uniform flow is not expected, the objective function will not be exactly zero.

The second case involved a straight wall contraction, where the design variable was the length of the contraction. There were 9 nodes across the channel. If the contraction length is the proper distance, then the waves created at the beginning of the contraction will strike the opposite wall at the end of the contraction and cancel out with the waves formed at the end of the contraction, producing uniform flow, as shown in Figure #1.

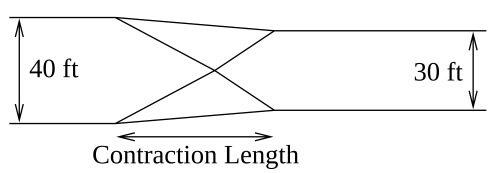

Figure #1. Overview of Channel Showing the Oblique Waves.

From the analytic equations governing an oblique hydraulic jump, the optimal contraction length is approximately 155.65 ft for the parameters chosen for these test cases. After 10 iterations of the design process, the contraction length was 157.870 ft. Since different assumptions are made in the analytic equations and in the shallow water equations, a small difference in the length of the contraction is expected.

The third test case was the same straight wall channel contraction problem as in the second test case but the grid was more refined. Instead of 9 nodes across the channel, there were 21 nodes. After 10 iterations, the contraction length for the more refined grid was 154.48 ft, which is close to the other lengths.

Two Design Variable Contraction Problem

For this problem, a channel contraction consisting of two straight walls was chosen. The contraction length was fixed at 140.0 ft, which was shorter than the optimal length for a straight wall contraction. The two design variables controlled the location of the junction between the two reaches, as shown in Figure #2. These design variables were modified to obtain uniform flow.

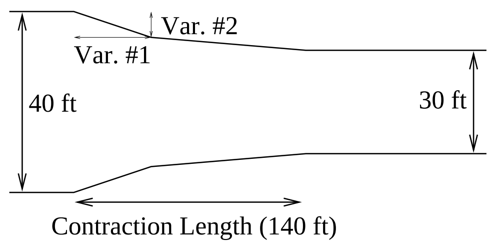

Figure #2. Overview of Channel Showing the Two Design Variables.

In this problem, the junction between the two reaches initially was located 56.0 ft downstream from the beginning of the contraction and 2.0 ft across the flow from the location of the initial channel width. Since these values were of different magnitudes, scaled design variables were used, so that both design variables were 2.0 initially. For these initial values, the objective function was 0.4166923. After 20 iterations, the objective function was 0.0550498 for design variables of 22.7765 ft and 0.2429 ft, respectively, a reduction of 86.8%. The design iteration process was not converged at this point, but the value of the objective function did not vary significantly for further iterations.

Three Design Variable Contraction Problem

This example used the same polynomial curved wall as the first test case, where the three design variables determined the polynomial. The channel contracts from 40 ft to 20 ft over a distance of 100 ft. Since the goal was to produce uniform flow, the objective function of eq. (7) was used. For comparison, a straight wall contraction of these dimensions produced a function value of 2.235.

After many executions of the code, using different initial design variables, it was determined that there was a local minimum near (3.49, 5.77, 8.30), yielding an objective function value of 0.316, or an 85.9% improvement. The probable global minimum was near (4.76, 7.66, 10.37) and gave a function value of 0.007, or a 99.6% improvement over the straight wall contraction.

Conclusion

From this preliminary investigation into using the adjoint variable formulation of direct differentiation to design high-velocity channels, the results are encouraging. The implementation using HIVEL2D demonstrates that it is possible to use this technique with a finite element solver and for the shallow water equations, both of which had not been previously investigated with this method.

From the viewpoint of an efficient design algorithm, the adjoint variable formulation is much better than a finite difference formulation of the gradient due to the need for only one steady state solution. Unfortunately, the method of steepest descent is not an efficient optimization algorithm as demonstrated by the large number of design iterations. This research will be extended to investigate other gradient based optimization algorithms.

The three verification cases presented in this paper were formulated such that an exact solution for the design problem was known. In these cases, the design optimization technique correctly identified a close approximation to the solution. For the last two examples, the optimal solution was not known. Nevertheless, for the two design variable problem, the technique demonstrated a 86.8% improvement over the initial design. And for the three design variable example, the optimal design was 99.6% better than the straight wall contraction.

Hence, with simple geometries, the adjoint variable formulation of direct differentiation yields design space gradients that can be used to calculate improved or optimized designs. Computational cost is proportional to the solution of one linear system of equations for each objective function, which is significantly less than the the cost of the corresponding finite difference approximations. This research is currently being extended for use with more complex configurations with the purpose of demonstrating a viable, deterministic design method applicable to flood control devices of general interest.

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