Classifying Improper Interval Graphs

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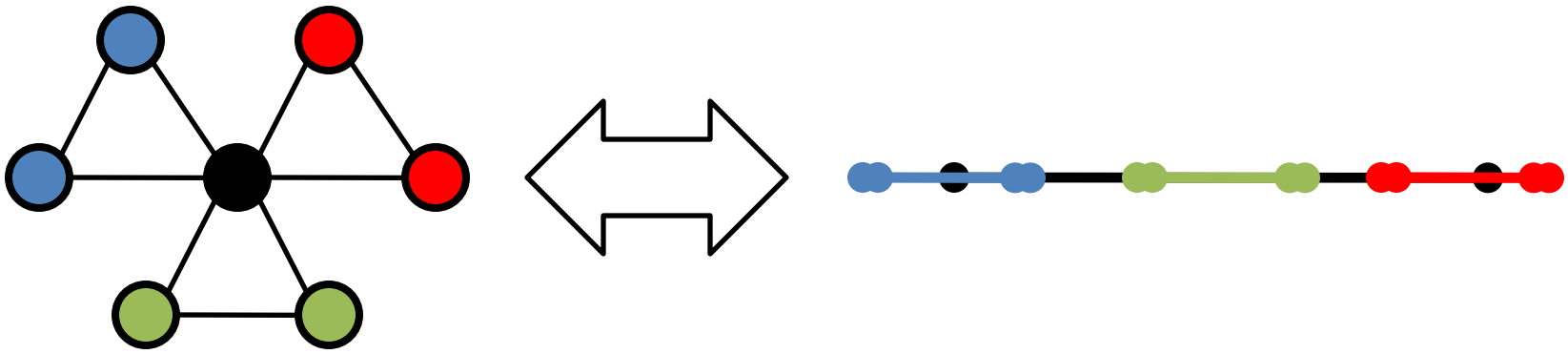
Interval Graphs

- Definition: A graph whose vertices may be represented as a set of closed intervals: where an edge occurs iff the corresponding intervals intersect
- (Equivalently: the intersection graph of closed intervals)



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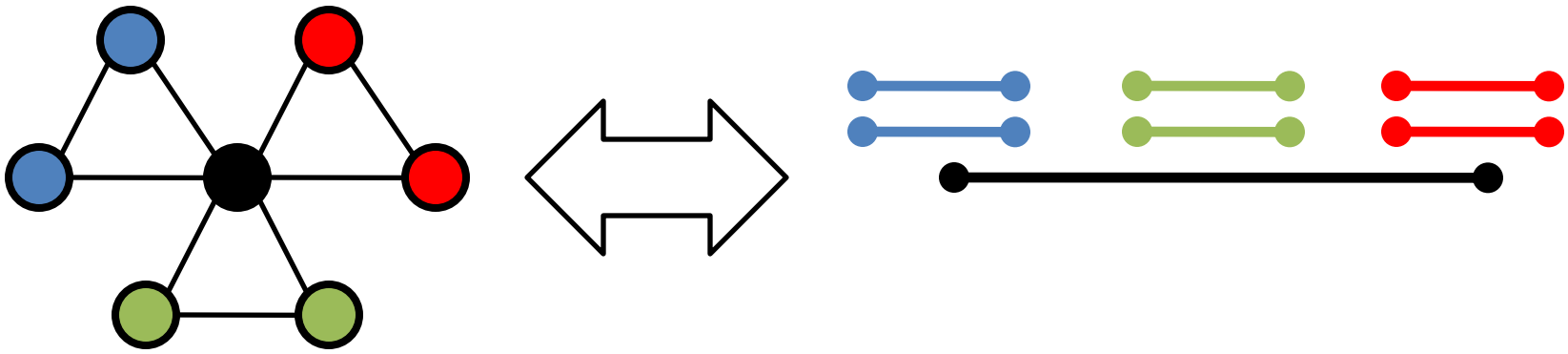
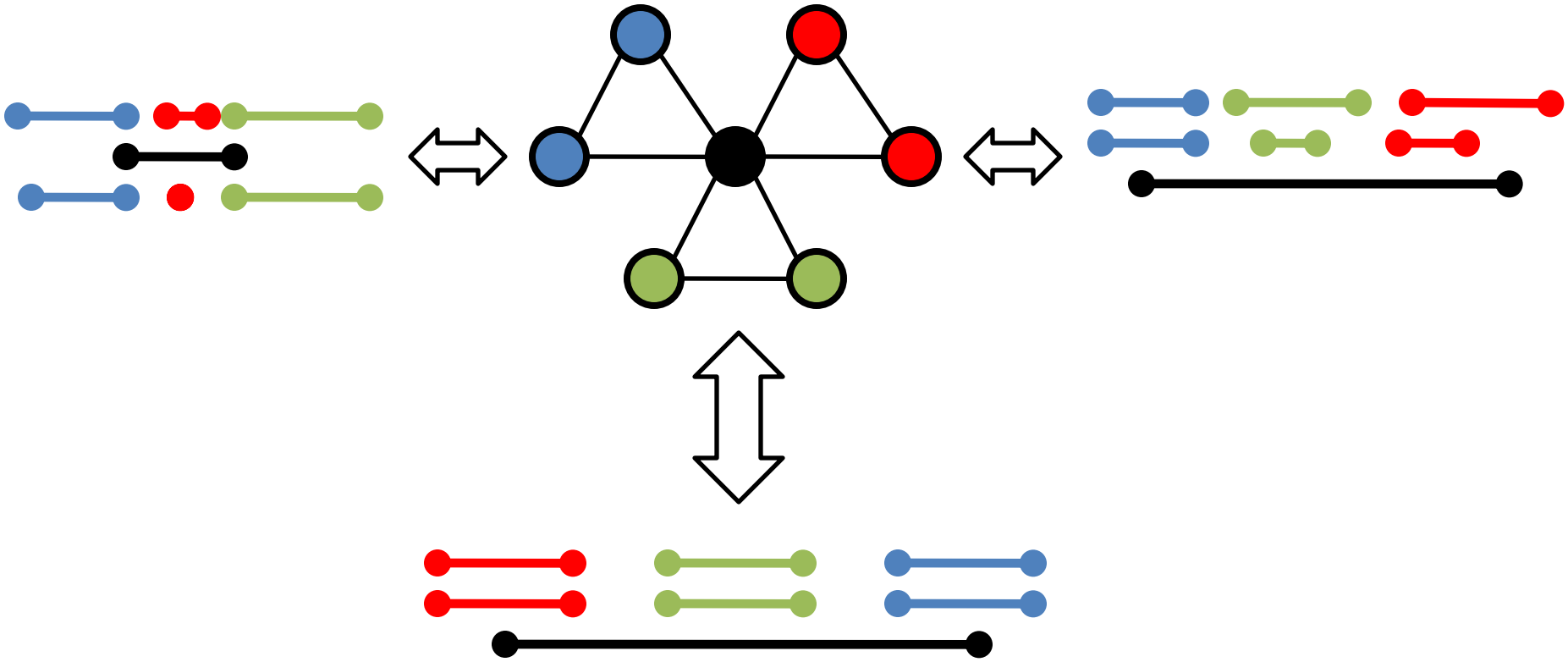
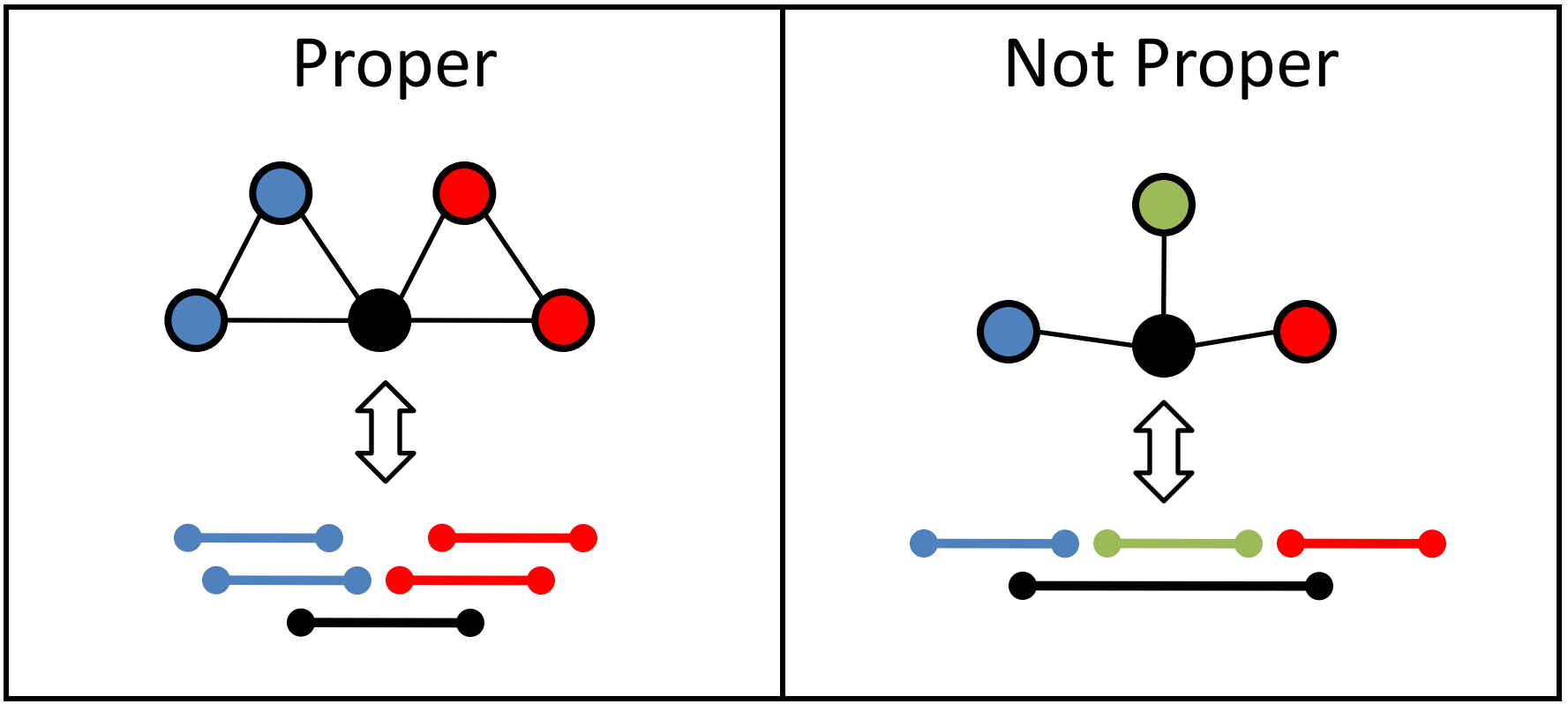


Nonuniqueness (of representation)



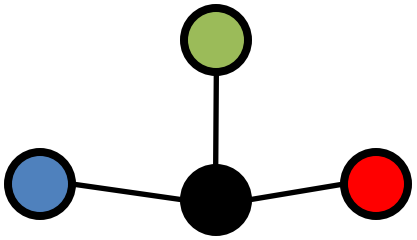
Proper Interval Graph

- Definition: An interval graph which has a representation in which no interval contains another.



Proper Interval Graphs

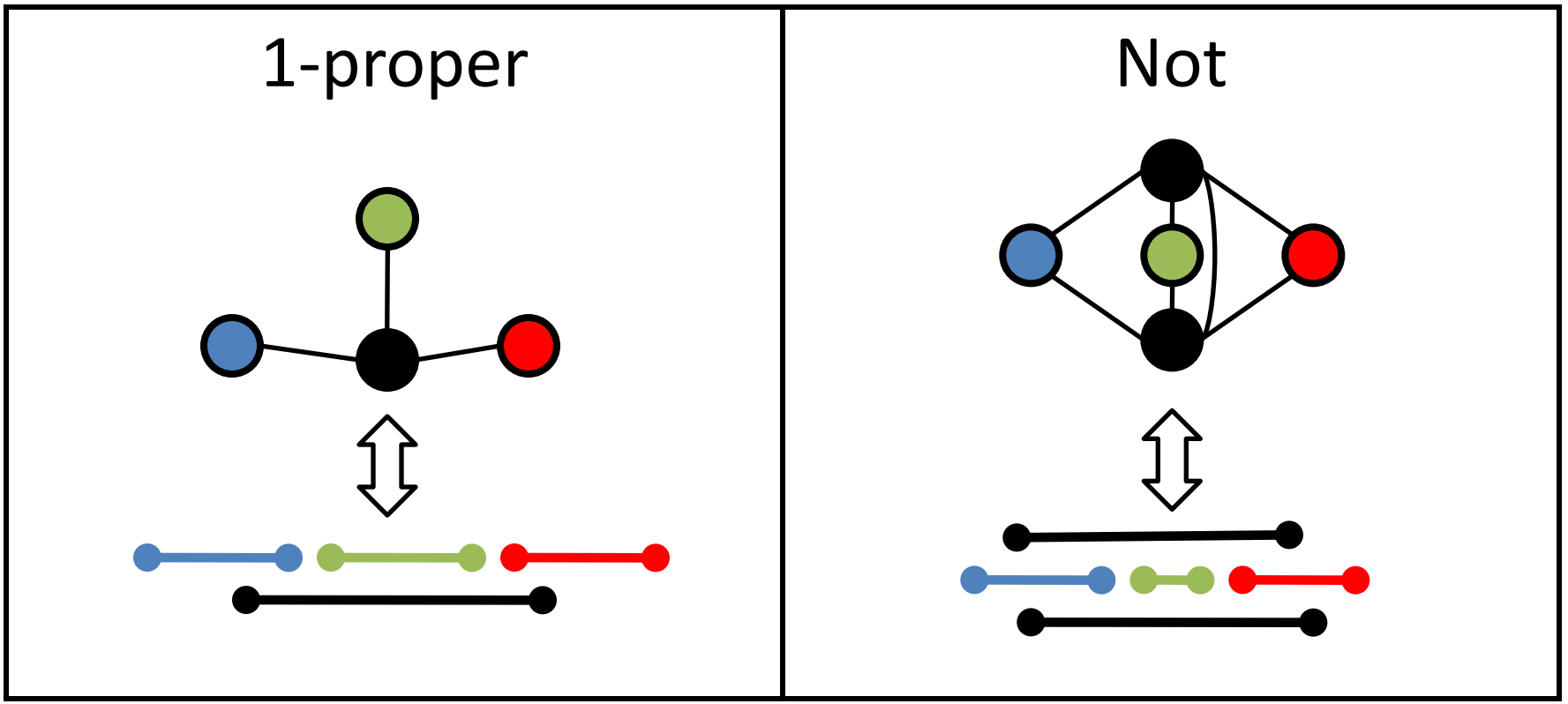
- Characterized in 1969 by Fred Roberts (Right)
- Characterization: an interval graph is proper iff it has no induced $K_{1,3}$



$K_{1,3}$ (aka claw, 3-star)

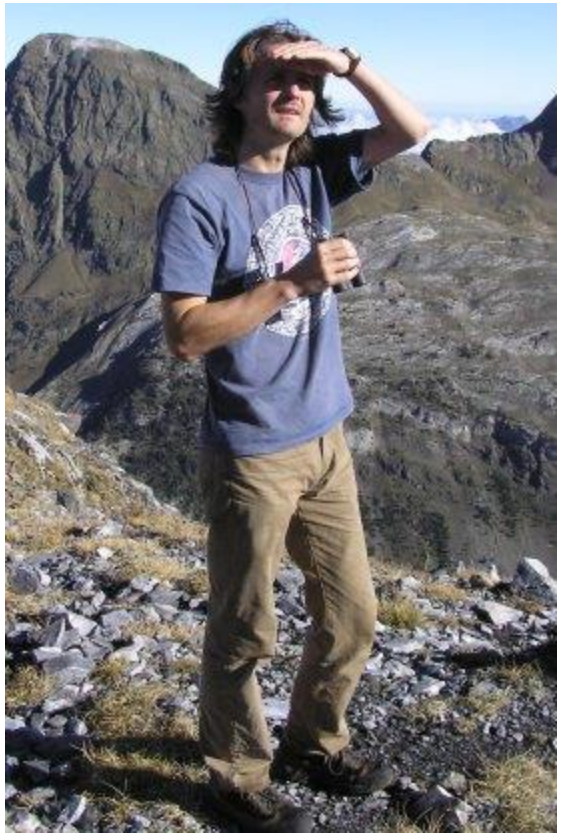
q-Proper Interval Graph

- Definition: An interval graph in which no interval is contained by more than *q* others.



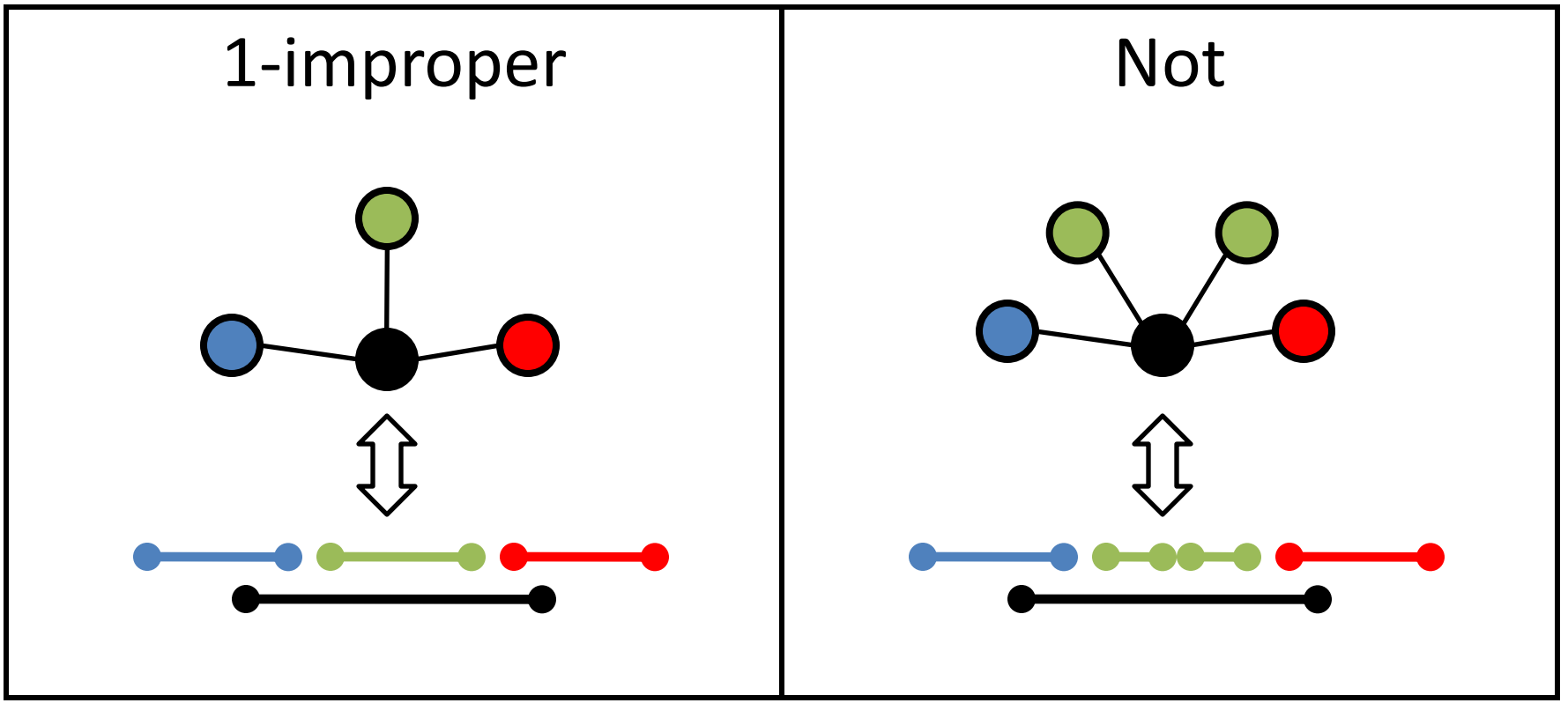
q-Proper Interval Graphs

- Characterized in 1999 A. Proskurowski (left) and J.A. Telle (right)
- Characterization: an interval graph is q-proper iff it has no induced T_{q+1}. (Tq+1 is a q+1 clique and three independent vertices, each one of which is adjacent to every vertex in the clique.)



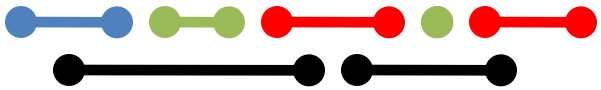
p-Improper Interval Graph

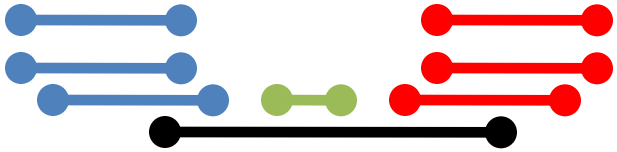
- Definition: An interval graph in which no interval contains more than *p* others.

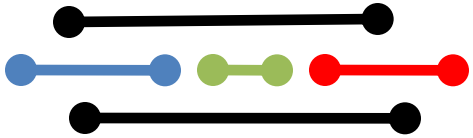


Examples

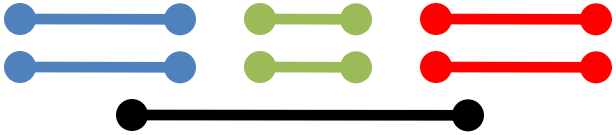
1-improper







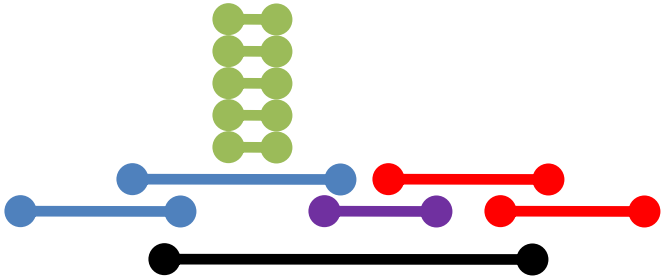
2-improper



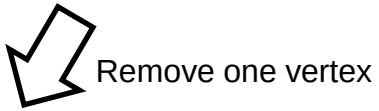
p-Improper Interval Graph

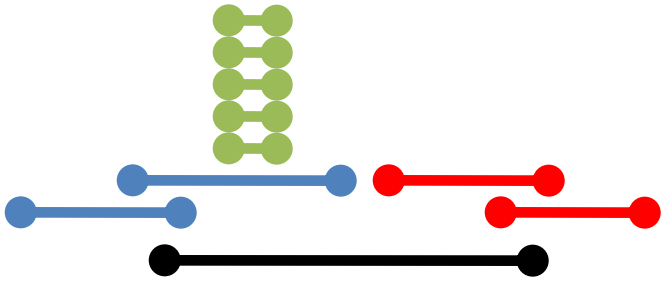
- Studied in 2008 by R. Jamison (right) and J. Beyerl.
- Classified 1-improper interval graphs.
- Partially classified balanced improper interval graphs

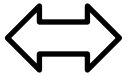
Instability

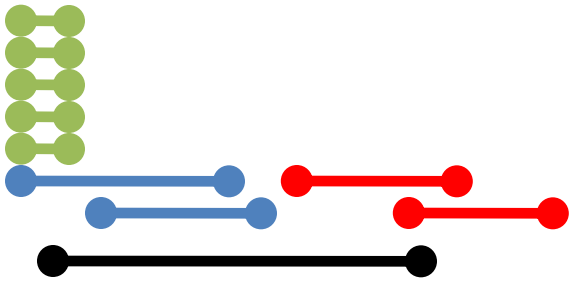


6-Improper









1-Improper

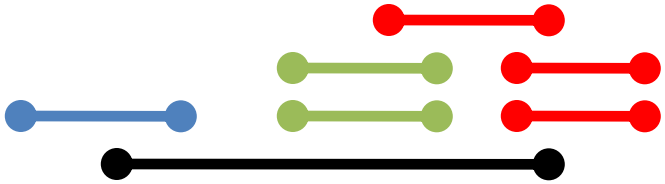
Forbidden Subgraphs

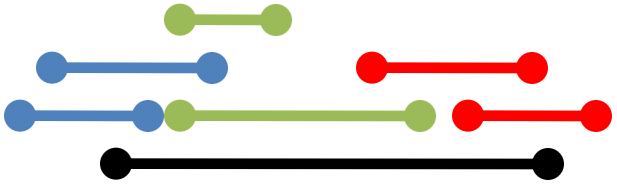
- Definition: A graph *H* is said to be forbidden if *H* cannot be an induced subgraph of *G*

- Such an H is said to be minimal if every proper subgraph of H is not forbidden

Examples of Forbidden Subgraphs

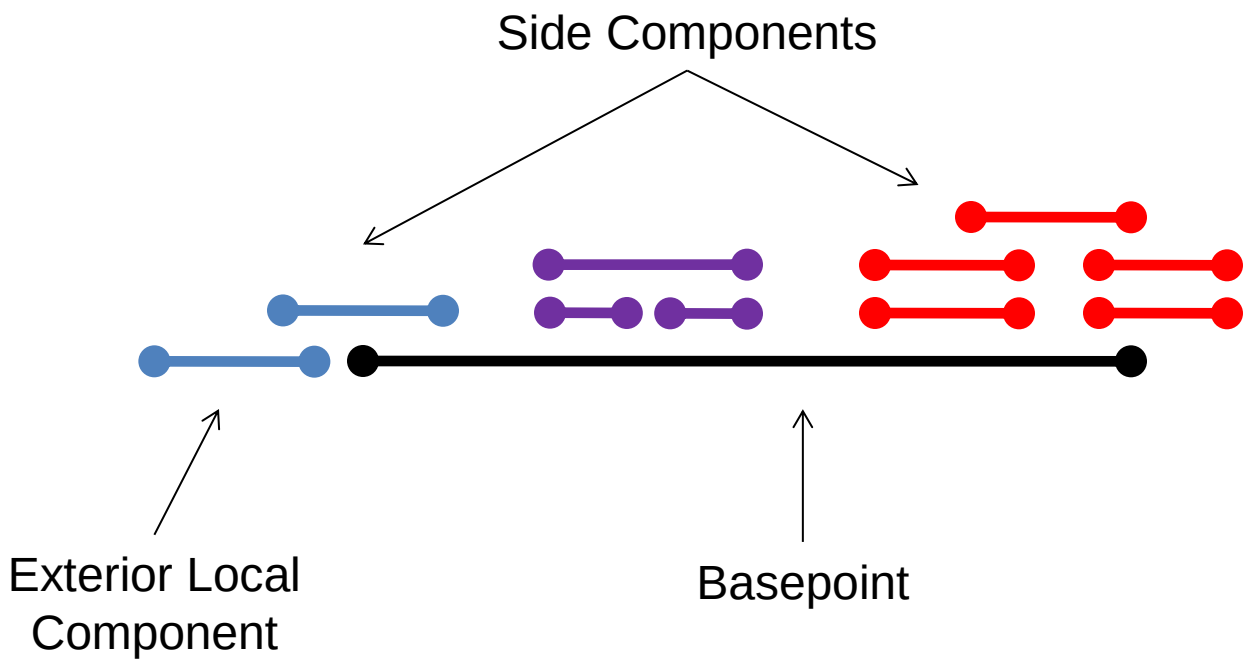
(For the class of 1-improper interval graphs)





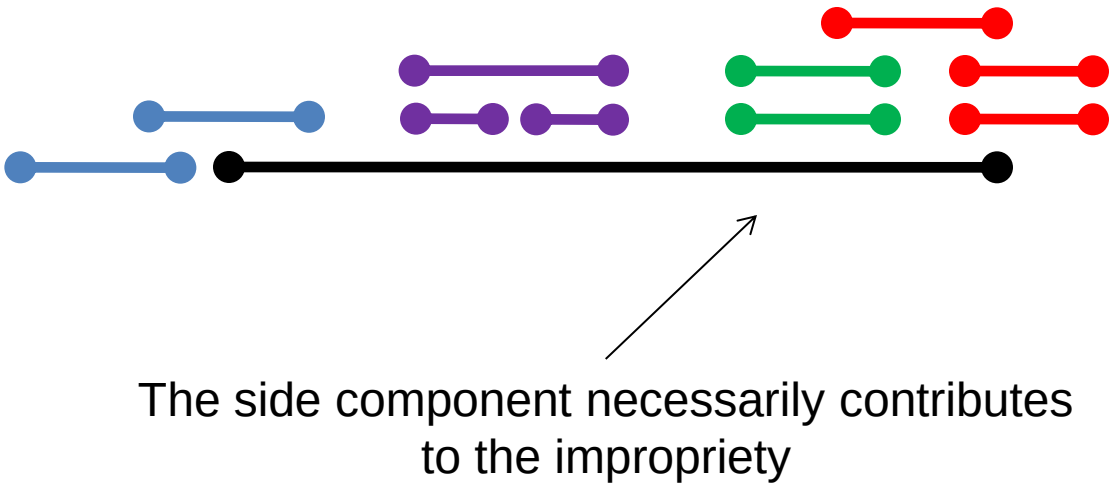
Local Components

- Given a basepoint, exterior local components are in the same place in every representation.



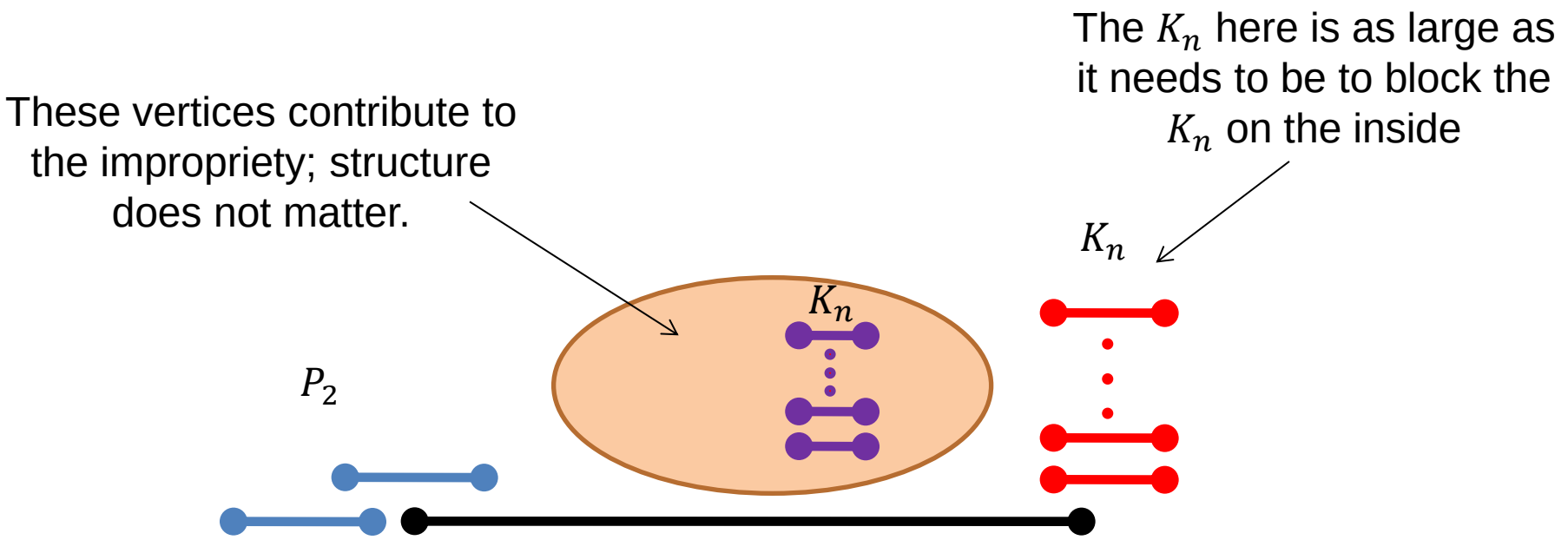
Balance

- A local component is considered balanced if it does not contribute to the impropriety when it is represented as a side component.



MFISGs with balanced side components

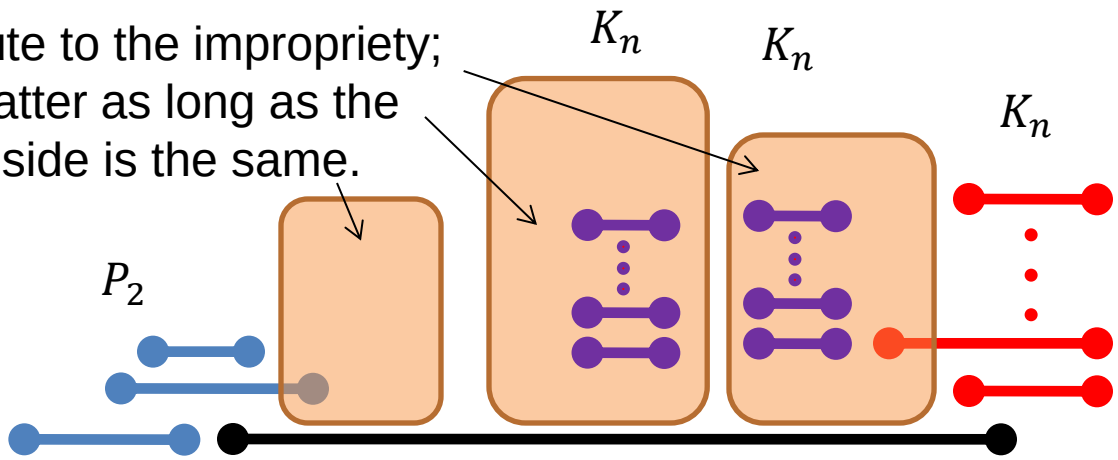
- Exterior local components are P_2
- Non-exterior local components are K_n



MFISGs with an unbalanced side component

- Exterior local components are P_2
- Non-exterior local components are K_n

These vertices contribute to the impropriety; structure does not matter as long as the potential K_n on the side is the same.



Applications

- Counting minimal forbidden interval subgraphs
- Comparing minimal forbidden interval subgraphs and critical graphs

Thank you!

Eratta

- It is relevant to note that after the talk we decided not to call balanced/unbalanced side components as such and to instead create the new terminology unconfined/confined.