The Singled Out Game

KENNAN SHELTON
Rhodes College
Memphis, TN 38112
shelton@rhodes.edu

In the mid 1990s, the cable music video channel MTV produced *Singled Out*, a game show in which fifty men competed to win a date with a single woman (for gender equity, the second half of the show featured fifty women vying for one man). Through a series of elimination stages—most of them embarrassing to the contestants—the field of potential suitors was reduced to only three.

To determine the winner from the final three contestants, the woman was presented with a series of either/or questions. Each man attempted to guess the woman's answer and those who guessed correctly took one step toward the woman. The winner was the man to take five steps and reach the woman. (A tie-breaker round was used if necessary.) Much of the time the men seemed to have no idea what answers the woman chose. I myself had no clue what she might answer—it seemed to me that she was simply flipping a coin to make her decision.

The interesting feature of the show was how the men announced their guesses. They were arranged in a fixed order and each in turn announced his guess so that the others could hear. Thus it was possible for the second and third men to modify their choices based on what the previous announcements had been. This raises the question: do the second and third men have any advantage, even if no man has any clue what the woman might answer? If so, what strategy could he employ to exploit this advantage? Would a little knowledge of mathematics help him to win?

The Singled Out game described in this article is an abstraction of the final stage of the MTV game show:

> *The Singled Out Game.* The Singled Out game is played in a series of rounds. In each round, a coin is flipped and the result is kept secret. Each player, one at a time, announces his guess so that the other players hear it. The order in which the players announce their guesses is determined at the start of the game and remains fixed throughout. After all players have guessed, the coin is revealed and all players who guessed correctly earn one point; incorrect guesses earn no points. The game continues until a player reaches n points and is declared the winner. If there is a tie, the winner is determined by a random selection from among those who tied.

In this paper we investigate the Singled Out game with two and three players. For the two-player game, the second player does indeed have an advantage—we obtain an expression for the probability that the second player wins and show that this probability approaches 1 as n approaches infinity. We then look at what happens if the first player is a good guesser (he is correct with some probability greater than 1/2) and show that this gives him at least a constant nonzero chance of winning, no matter how many points are needed to win. Finally, we look at the three-player game and present experimental evidence that the third player does have an advantage, though his strategy is counter-intuitive.

The analysis of the two-player Singled Out game involves *Catalan numbers*, a well-known sequence, which we briefly describe here. Catalan numbers may be used to enumerate a wide variety of objects. Euler first described them in the 18th century when he investigated the number of triangulations of convex polygons; Catalan used them one hundred years later to count the number of binary parenthesizations of a string. Over 60 applications of Catalan numbers are given in [**5**]. For more information about Catalan numbers, including an extensive bibliography, see [**3**]. In this paper we will focus on one application of Catalan numbers: the enumeration of *ballot paths*.

In a two-dimensional integer lattice, consider the paths from $(0, 0)$ to (n, k), $n \geq k \geq 0$, that move either horizontally to the right or vertically up by one-unit steps. Ballot paths are those paths that never rise above the diagonal, that is, for all (a, b) in the path, we have $a \geq b$. For example, there are 5 ballot paths from $(0, 0)$ to $(3, 3)$ (see FIGURE 1).

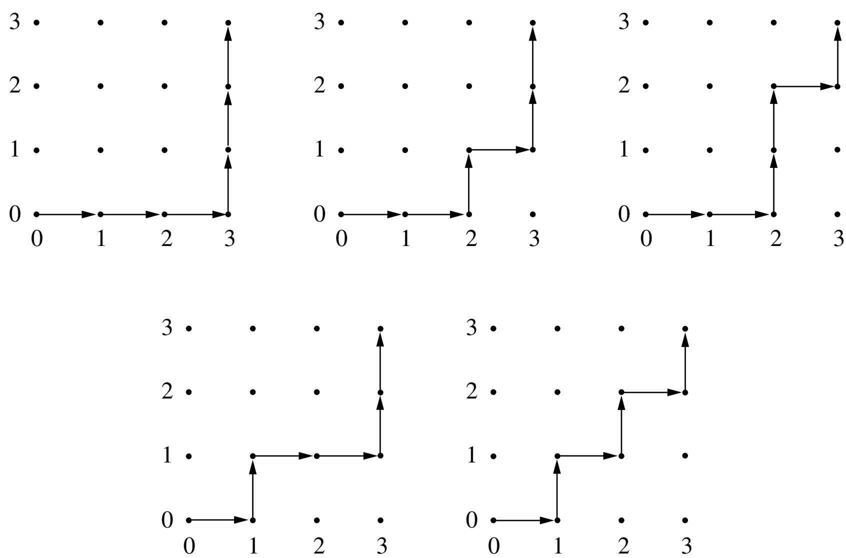

Figure 1 The five ballot paths to $(3, 3)$

The term "ballot path" comes from the role these paths play in solving the *Ballot Problem*. Suppose that candidates A and B receive n and k votes respectively, with $n \geq k$. The Ballot Problem asks for the probability that, as we count the ballots, A is never behind B in the tally. This is equivalent to asking what proportion of all paths from $(0, 0)$ to (n, k) are ballot paths. The Ballot Problem was originally solved by Bertrand; an elegant solution was given by André using what is now referred to as André's Reflection Principle [**1**]. We should note that the original problem considered by Bertrand and André asked for the probability that the count for A is *always ahead* of B (this probability is $(n - k)/(n + k)$). For our purposes, however, it is more convenient to allow A and B to tie during the tally.

Let $C(n, k)$ denote the number of ballot paths from $(0, 0)$ to (n, k), $n \geq k \geq 0$. It is easy to see that the $C(n, k)$ satisfy the recurrence equations:

$$C(n, k) = C(n - 1, k) + C(n, k - 1), \qquad n > k \geq 1;$$
$$C(n, n) = C(n, n - 1), \qquad n \geq 1;$$
$$C(n, 0) = 1, \qquad n \geq 0.$$

The reader is invited to show that $C(n, k)$ has the closed form expression

$$C(n, k) = \binom{n+k}{k} - \binom{n+k}{k-1}. \tag{1}$$

The nth Catalan number, C_n, is the number of ballot paths from (0, 0) to (n, n), i.e., $C_n = C(n, n)$. The first several Catalan numbers are 1, 1, 2, 5, 14, and 42. Using (1) we see that

$$C_n = \frac{1}{n+1}\binom{2n}{n}. \tag{2}$$

A recurrence relation and an alternative expression for C_n will also be useful to us. These can be derived easily from (2).

$$C_n = \frac{2(2n-1)}{n+1}C_{n-1}, \quad C_0 = 1; \tag{3}$$

$$C_n = \frac{2^n}{(n+1)!}\prod_{k=1}^{n}(2k-1) = \frac{4^n}{(n+1)!}\prod_{k=1}^{n}(k-1/2), \quad n \geq 1. \tag{4}$$

Two-player Singled Out

Suppose that we only have two players, Adam and Barry, who are playing to n points and that Adam always guesses first. We will assume in this section that Adam, having no information at all, has probability $1/2$ of guessing correctly. Barry, on the other hand, does have some information: he knows what Adam has guessed. We will see that this extra piece of information is enough to give Barry the edge with the right strategy.

A play of the game may be represented as a *game path* on a two-dimensional integer lattice. The integer point (a, b), $0 \leq a, b$, represents the game state in which Adam has a points and Barry has b points. Then a game to n points can be represented as a path from (0, 0) to one of the points (n, k) or (k, n) for $0 \leq k \leq n$. Note that if $k = n$ then the game ends in a tie and the winner is decided randomly.

Since the players will either gain a point or not, a path from (0, 0) to a winning state will move in one of three nontrivial ways: *right* if Adam guesses correctly and Barry guesses incorrectly, *up* if Adam guesses incorrectly and Barry guesses correctly, and *diagonally* if Adam and Barry both guess correctly. In the case that both Adam and Barry guess incorrectly, the game state remains the same. An example game path is presented in FIGURE 2, where Adam has won with 6 points versus 5 points for Barry.

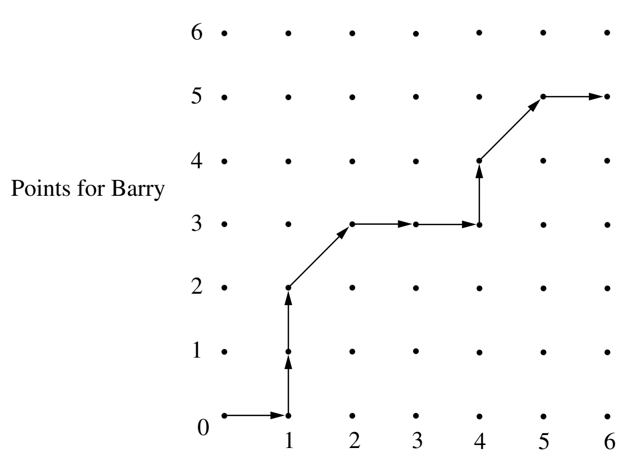

Figure 2 A winning game path for Adam

Two simple strategies that Barry may employ are the *copy-cat* strategy (always make the same guess as Adam) and the *contrarian* strategy (always make the opposite guess as Adam). However, it is not hard to show that either of these strategies will give Barry a probability of 1/2 of winning. Thus they confer no advantage to Barry.

But Barry can actually increase his probability of winning to greater than 1/2 by the following strategy:

> *Contrari-cat strategy*: Whenever Barry is behind or tied with Adam, he should choose the opposite of Adam. Whenever Barry is ahead of Adam, he should choose the same as Adam.

Using the contrari-cat strategy, if Barry ever gets ahead of Adam then Barry will be guaranteed to win as he will always be one point ahead of Adam. The probability that Barry gets ahead on the first flip of the coin (and then wins the game) is 1/2, so this strategy is at least as good as the copy-cat and contrarian strategies. If they are playing to only one point then Barry cannot do any better.

However, if the game requires two or more points to win, Barry will have a nonzero probability of catching up to Adam and then getting ahead. Thus the probability that Barry wins is greater than 1/2 and he has the advantage. For example, suppose that the game is being played to 3 points. For Barry to win using the contrari-cat strategy, one of the following situations must occur:

1. Barry gets ahead on the first flip (from (0, 0) to (0, 1))
2. Barry loses the first flip, but catches up on the second and gets ahead on the third (from (1, 1) to (1, 2))
3. Barry loses two flips (the first two or the first and third) but then catches up and gets ahead on the fifth flip (from (2, 2) to (2, 3)).

The possible game paths are shown in FIGURE 3. Note that there are two winning game paths passing through (2, 2), each with equal probability (1/32) of occurring. Thus the probability that Barry wins will be $1/2 + 1/8 + 2/32 = 11/16$.

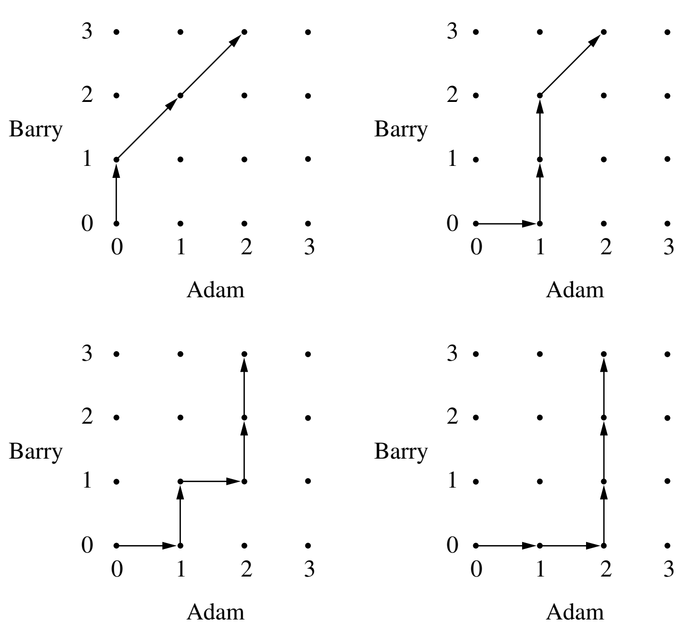

Figure 3 Barry's winning game paths to 3 points

We want to determine the probability $P_B(n)$ that Barry wins when playing to n points using the contrari-cat strategy. In general, Barry will win if at any time the game path crosses above the diagonal from $(0, 0)$ to (n, n), since he will be ahead at this point and his strategy will prevent Adam from winning. Thus winning game paths are in one-to-one correspondence with the set of ballot paths from $(0, 0)$ to (k, k), $0 \le k \le n - 1$.

Suppose G is a winning game path for Barry that first crosses above the diagonal at (k, k) for some k between 0 and $n - 1$. The path G must contain k moves to the right and k moves up, followed by one more move up (to cross the diagonal). All moves occur with probability 1/2, so the probability that G occurs is $1/(2 \cdot 4^k)$. However, there are C_k such paths, so the probability that Barry wins by crossing at (k, k) is $C_k/(2 \cdot 4^k)$. Therefore

$$P_B(n) = \frac{1}{2}\sum_{k=0}^{n-1} \frac{1}{4^k} C_k.$$

Since this formula shows that $P_B(n + 1) > P_B(n)$, as the number of points required to win the game increases, Barry has a better and better chance of winning when using the contrari-cat strategy. We can use this fact to show that the contrari-cat strategy is better than any other. For clearly if Barry is ahead then he should follow the copycat strategy and if he is behind then he should follow the contrarian strategy (in order to win, Barry must say the opposite of Adam at some point; the earlier he does this, the greater his probability of catching up before Adam wins the game). We only need to determine what Barry should do at the start of the game, when the score is tied. If Barry did not follow the contrarian strategy at this point then the probability that he wins will be the same as if Adam and Barry were playing to one fewer point. But the fact that $P_B(n)$ is a strictly increasing function of n, coupled with a simple induction argument, shows that Barry would be better off playing the contrarian strategy. Thus the contrari-cat strategy is optimal for Barry—no other strategy can guarantee a higher probability of winning.

In fact, as n approaches infinity, $P_B(n)$ approaches 1, as we show next.

THEOREM 1. *As the two-player Singled Out game is played to an increasing number of points, the probability that the second player wins using the contrari-cat strategy approaches* 1, *that is,*

$$\lim_{n\to\infty} P_B(n) = 1.$$

Proof. From the definition of $P_B(n)$, we have that

$$P_B(n) = \frac{1}{2}\sum_{k=0}^{n-1} \frac{1}{4^k} C_k = P_B(n-1) + \frac{1}{2 \cdot 4^{n-1}} C_{n-1}.$$

Using induction, we will first show that for all $n \ge 1$,

$$P_B(n) = 1 - \frac{n+1}{4^n} C_n. \qquad (5)$$

Inspection shows that (5) holds when $n = 1$. Suppose that (5) holds for some $n \ge 1$. Using the recursive expression (3) for the Catalan numbers we have

$$\begin{aligned} P_B(n+1) &= P_B(n) + \frac{1}{2\cdot 4^n}C_n \\ &= 1 - \frac{n+1}{4^n}C_n + \frac{1}{2\cdot 4^n}C_n = 1 - \frac{2n+1}{2\cdot 4^n}C_n \\ &= 1 - \frac{2n+1}{2\cdot 4^n}\cdot\frac{n+2}{2(2n+1)}C_{n+1} = 1 - \frac{n+2}{4^{n+1}}C_{n+1}, \end{aligned}$$

so by induction (5) holds for all $n \geq 1$.

Then, using the alternative expression (4) for C_n, we have

$$\begin{aligned} P_B(n) &= 1 - \frac{n+1}{4^n}\cdot\frac{4^n}{(n+1)!}\prod_{k=1}^{n}(k-1/2) \\ &= 1 - \frac{\prod_{k=1}^{n}(k-1/2)}{\prod_{k=1}^{n}k} \\ &= 1 - \prod_{k=1}^{n}\left(1-\frac{1}{2k}\right). \end{aligned}$$

Thus to show that $P_B(n)$ approaches 1, it suffices to show that the infinite product

$$\prod_{k=1}^{\infty}\left(1-\frac{1}{2k}\right) \tag{6}$$

converges to 0. Let $p_n = \prod_{k=1}^{n}(1-1/(2k))$. Since $\{p_n\}$ is a decreasing sequence bounded below by 0, it must converge and so (6) exists. The infinite product converges to a nonzero number if and only if the series

$$\sum_{k=1}^{\infty}\ln\left(1-\frac{1}{2k}\right)$$

converges [**2**, p. 164]. However, using the power series expansion of $\ln(1-x)$,

$$\ln(1-x) = -x - \frac{x^2}{2} - \frac{x^3}{3} - \cdots \qquad |x| < 1,$$

we see that for $k \geq 1$, $\ln(1-1/2k) \leq -1/2k$. Thus the series cannot converge and so (6) converges to 0. Therefore $\lim_{n\to\infty} P_B(n) = 1$. ■

As a corollary to Theorem 1, we have the following identity that will be used later:

$$\sum_{k=0}^{\infty}\frac{1}{4^k}C_k = 2.$$

Remark Alternatively, we might have shown that the probability $P_A(n)$ that Adam wins, converges to 0 as n approaches infinity. Winning paths for Adam are precisely the ballot paths from $(0, 0)$ to (n, k) with $n > k \geq 0$. The reader is invited to use the recurrence properties of $C(n, k)$ to show that

$$P_A(n) = \frac{1}{2^n}\sum_{k=0}^{n-1}\frac{1}{2^k}C(n-1, k)$$

$$= \frac{2n-1}{2n} P_A(n-1)$$
$$= \prod_{k=1}^{n}\left(1 - \frac{1}{2k}\right),$$

which we know converges to 0 as n approaches infinity.

Arbitrary probabilities

In the previous section we assumed that Adam's chance to guess correctly was the same as flipping a fair coin. But what if he is a good guesser? In terms of the MTV game, perhaps Adam has some insight into the psyche of the woman who is answering the questions. Can Adam overcome the advantage Barry has with the contrari-cat strategy? We will show in this section that if Adam has better than even chance to guess correctly, then he will have at least a constant nonzero chance to win, independent of how many points are required. We will assume that Barry has no knowledge of Adam's ability to guess correctly and that he will continue to follow the contrari-cat strategy.

Suppose that Adam is correct with probability $1/2 + t$ for some $t \in [-1/2, 1/2]$ (so he is wrong with probability $1/2 - t$). Barry's contrari-cat strategy will still guarantee a win provided he can get ahead of Adam. But the probability that Barry will get ahead when playing to n points is now a function of t as well as n.

Let $P_B(n, t)$ denote the probability that Barry wins the game when playing to n points and when Adam is correct with probability $1/2 + t$. Thus $P_B(n, 0) = P_B(n)$ as defined in the previous section. Note that $P_B(n, -1/2) = 1$ while $P_B(n, 1/2) = 0$ for all $n \geq 1$. Suppose that $t \in (-1/2, 1/2)$. As before, Barry will win when a game path rises above the diagonal. For each k between 0 and $n - 1$, to reach the state (k, k) the game path will need to make k moves to the right and k moves up, followed by one more move up. However, now the probability of a move to the right is $1/2 + t$ while the probability of moving up is $1/2 - t$. Thus

$$P_B(n,t) = \left(\frac{1}{2} - t\right) \sum_{k=0}^{n-1} \left(\frac{1}{2} - t\right)^k \left(\frac{1}{2} + t\right)^k C_k \tag{7}$$

$$= \left(\frac{1}{2} - t\right) \sum_{k=0}^{n-1} \left(\frac{1}{4} - t^2\right)^k C_k. \tag{8}$$

The probability $P_B(n, t)$ is again an increasing function of n for fixed t, because the terms in the sum are positive. Also, $P_B(n, t)$ is a polynomial function of t with $dP_B(n, t)/dt < 0$ so it is a decreasing function of t for fixed n.

We will find an explicit expression for the function $P_B^*(t) = \lim_{n\to\infty} P_B(n, t)$. Note that for $t \leq 0$, it is clear that $P_B^*(t) = 1$ since $P_B(n, t) \geq P_B(n, 0)$ for each $n \geq 1$ and $P_B(n, 0) = P_B(n) \to 1$ by Theorem 1.

For $t \in [-1/2, 1/2]$, set

$$S(t) = \begin{cases} 1 & \text{if } t = \pm 1/2 \\ \displaystyle\sum_{k=0}^{\infty} \left(\frac{1}{4} - t^2\right)^k C_k & \text{otherwise.} \end{cases}$$

Note that $S(t) = S(-t)$ and $S(0) = \sum_{k=0}^{\infty} C_k/4^k = 2$. Since $0 \leq (1/4 - t^2)^k \leq 1/4^k$ for all $k \geq 1$, $S(t)$ exists and is at most 2. Further, $P_B^*(t) = (1/2 - t)S(t)$.

Now, for $t \in [-1/2, 0]$, $P_B^*(t) = 1$ so $S(t) = 2/(1-2t)$. Then $S(t) = 2/(1+2t)$ for $t \in (0, 1/2]$. Therefore

$$P_B^*(t) = \begin{cases} 1 & \text{if } -1/2 \le t \le 0 \\ \dfrac{1-2t}{1+2t} & \text{if } 0 < t \le 1/2. \end{cases}$$

Thus for $t > 0$, the probability that Adam wins is at least $1 - (1-2t)/(1+2t) = 4t/(1+2t)$, for all values of n.

Note that $4t/(1+2t) = 1/2$ when $t = 1/6$. So for large n, for Adam to have an approximately even chance of winning against Barry (who is assumed to be using the contrari-cat strategy), he will need to guess correctly at least $1/2 + 1/6 = 2/3$ of the time.

Three-player Singled Out

We now turn to experimental results for the three-player game, with players Adam, Barry, and Carl. As with the original two-player game, Adam has no information—his probability of guessing correctly is 1/2. Barry has the same information as before (Adam's choice) and Carl has the most information available to him before he makes his choice (Adam's and Barry's choices).

In this section, we will represent the game states as 3-tuples (a, b, c) where Adam needs a more points to win, Barry needs b more points and Carl needs c more points. The game ends when at least one of a, b, or c is zero. Recall that if there is a tie then we will choose randomly to determine the winner.

Let $P(a, b, c)$ be the 3-tuple denoting the probabilities that Adam, Barry, and Carl (respectively) will win when starting in game state (a, b, c). We will determine $P(a, b, c)$ recursively using the initial values shown in TABLE 1.

TABLE 1: Initial winning probabilities

(a, b, c)	$P(a, b, c)$	(a, b, c)	$P(a, b, c)$
(0, 0, 0)	(1/3, 1/3, 1/3)	(1, 0, 0)	(0, 1/2, 1/2)
(0, 0, 1)	(1/2, 1/2, 0)	(1, 0, 1)	(0, 1, 0)
(0, 1, 0)	(1/2, 0, 1/2)	(1, 1, 0)	(0, 0, 1)
(0, 1, 1)	(1, 0, 0)		

To calculate $P(1, 1, 1)$ we first determine the possible game state outcomes when starting from state (1, 1, 1). Since Adam has an even chance of winning a point no matter what he guesses, we focus on the choices of Barry and Carl. Each has two choices: to say the same as Adam (A) or not ($\neg A$). Since Adam is either correct or incorrect, there are 8 possible outcomes, summarized in TABLE 2. In each entry, the upper 3-tuple is the resulting outcome game state if Adam is correct; the lower 3-tuple is the outcome state if Adam is incorrect. We can then calculate the winning probabilities $P(1, 1, 1)$, shown in TABLE 3.

If Barry chooses A then Carl will certainly choose $\neg A$ as it gives him the greater chance of winning. If Barry chooses $\neg A$ then it does not matter what Carl chooses: he will have probability 1/4 of winning for either choice. However, Carl's choice will affect Barry's chance of winning. What should Carl do? If he is amiable and can forgive

TABLE 2: Game state outcomes for (1, 1, 1)

		Carl: A	Carl: $\neg A$
Barry	A	(0, 0, 0) (1, 1, 1)	(0, 0, 1) (1, 1, 0)
	$\neg A$	(0, 1, 0) (1, 0, 1)	(0, 1, 1) (1, 0, 0)

TABLE 3: Winning probabilities for (1, 1, 1)

		Carl: A	Carl: $\neg A$
Barry	A	(1/3, 1/3, 1/3)	(1/4, 1/4, 1/2)
	$\neg A$	(1/4, 1/2, 1/4)	(1/2, 1/4, 1/4)

Barry for limiting him to a winning probability of 1/4 then Carl will choose A; if he is vindictive and wants to take Barry down with him then he will choose $\neg A$.

To move forward in our analysis, we make the assumption that Barry and Carl each rank the players in order of preference of winner. Of course each player will rank himself first, leaving only two possibilities for their preferences: he is *Vindictive* if he prefers that Adam win over the other player or *Amiable* if he prefers that the other player win over Adam. In this way we change the original three player game to a two-player *preference game* between Barry and Carl in which the options are Amiable or Vindictive. The payoff matrix for the game is the resulting matrix of winning probabilities and each player is attempting to maximize his winning payoff (probability). TABLE 4 shows the payoff matrix for this game when one point is needed to win (the payoff/probability vectors are transposed).

TABLE 4: Values of $P(1, 1, 1)$ (payoff matrix) for the preference game

		Carl: Amiable	Carl: Vindictive
Barry	Amiable	$\begin{pmatrix} 0.25 \\ 0.5 \\ 0.25 \end{pmatrix}$	$\begin{pmatrix} 0.25 \\ 0.25 \\ 0.5 \end{pmatrix}$
	Vindictive	$\begin{pmatrix} 0.25 \\ 0.5 \\ 0.25 \end{pmatrix}$	$\begin{pmatrix} 0.5 \\ 0.25 \\ 0.25 \end{pmatrix}$

Once we have fixed the preferences of Barry and Carl, we can recursively determine the values of $P(n, n, n)$ for $n \geq 1$. Experimental evidence shows that for $14 \leq n \leq 100$, Barry is better off being vindictive while Carl is better off being amiable. In fact, for these values of n, the strategy of (Vindictive, Amiable) is a *pure Nash equilibrium*—neither player can improve his payoff by unilaterally changing his strategy [**4**, p. 128]. TABLE 5 shows the possible values of $P(100, 100, 100)$. It is not known if this pattern continues for $n > 100$.

TABLE 5: Values of $P(100, 100, 100)$ for the preference game

		Carl: Amiable	Carl: Vindictive
Barry	Amiable	$\begin{pmatrix} 0.0142 \\ 0.3816 \\ 0.6042 \end{pmatrix}$	$\begin{pmatrix} 0.1210 \\ 0.3229 \\ 0.5561 \end{pmatrix}$
Barry	Vindictive	$\begin{pmatrix} 0.0130 \\ 0.4113 \\ 0.5757 \end{pmatrix}$	$\begin{pmatrix} 0.1266 \\ 0.3337 \\ 0.5397 \end{pmatrix}$

In addition to being Vindictive and Amiable, Barry and Carl may also be *Indifferent*—they have no second choice for winner and will choose randomly between two options that give them the same probability outcome. With this extra option, there are nine possible values of $P(n, n, n)$. TABLE 6 shows the payoff/probability matrix $P(100, 100, 100)$.

TABLE 6: Values of $P(100, 100, 100)$ for the preference game

		Carl: Amiable	Carl: Indifferent	Carl: Vindictive
Barry	Amiable	$\begin{pmatrix} 0.0142 \\ 0.3816 \\ 0.6042 \end{pmatrix}$	$\begin{pmatrix} 0.1043 \\ 0.3497 \\ 0.5460 \end{pmatrix}$	$\begin{pmatrix} 0.1210 \\ 0.3229 \\ 0.5561 \end{pmatrix}$
Barry	Indifferent	$\begin{pmatrix} 0.0108 \\ 0.3971 \\ 0.5920 \end{pmatrix}$	$\begin{pmatrix} 0.1043 \\ 0.3497 \\ 0.5460 \end{pmatrix}$	$\begin{pmatrix} 0.0222 \\ 0.3742 \\ 0.6036 \end{pmatrix}$
Barry	Vindictive	$\begin{pmatrix} 0.0130 \\ 0.4113 \\ 0.5757 \end{pmatrix}$	$\begin{pmatrix} 0.1043 \\ 0.3497 \\ 0.5460 \end{pmatrix}$	$\begin{pmatrix} 0.1266 \\ 0.3337 \\ 0.5397 \end{pmatrix}$

Note that the Amiable choice for Barry is dominated by both the Indifferent and Vindictive choices. Also, the Indifferent choice for Carl is dominated by the Amiable choice. Thus the payoff matrix for the preference game can be pared down to only the Indifferent and Vindictive choices for Barry and the Amiable and Vindictive choices for Carl. In this reduced game there are *two* pure Nash equilibria, one at (Indifferent,Vindictive) and another at (Vindictive, Amiable).

In fact, if we rank the outcomes in order of preference of the players (where 1 is the least preferred and 4 is the most preferred), we obtain TABLE 7, the game of Chicken:

> Two adversaries are set on a collision course. If both persist, then a very unpleasant outcome, sometimes mutual annihilation, is guaranteed. If only one of the players swerves away (chickens) he loses the game. If both swerve, the result is a draw [**4**, p. 125].

TABLE 7: The game of Chicken

		Carl: Amiable	Carl: Vindictive
Barry	Indifferent	(3, 3)	(2, 4)
Barry	Vindictive	(4, 2)	(1, 1)

In our situation, the "very unpleasant" outcome occurs when both players are Vindictive—they work against each other and give each other the least probabilities to win. While Barry and Carl are still each more likely to win than Adam, they could do even better if they were able (and willing) to cooperate and play the (Indifferent, Amiable) strategy. More information on games like this, where there is no clear strategy for the players, can be found in Stahl [**4**] and Straffin [**6**].

Similar results hold for $5 \le n \le 100$ (except $n = 11$, 14, and 17). Further, for all n between 1 and 100, the values of $P(n, n, n)$, given an Indifferent Carl, are all equal. As before, general results are not known for $n > 100$.

Conclusion

In the two-player Singled Out game, the second player has the advantage by playing the contrari-cat strategy. Further, the probability that the second player will win increases to one as the number of points to win increases to infinity. We saw also that the first player can overcome this disadvantage provided that he is able to guess correctly better than $2/3$ of the time.

The more interesting case of three players is still open. Our calculations for small n indicate that, as expected, the third player does have an advantage. But this advantage does not seem to be overwhelming, and it also relies on the third player using the counter-intuitive strategy of working with the second player (who is his closest rival).

Several questions remain. In particular, when Barry and Carl are allowed to be Vindictive or Amiable, will there always be a Nash equilibrium at (Vindictive, Amiable) for large n? Will $P(n, n, n)$ approach a fixed payoff/probability vector? With the Indifferent choice, will the preference game reduce to one of Chicken for large enough n? What effect would communication have on the game? Is there another method of analyzing the three player game other than reducing it to a preference game? And what happens when we play the Singled Out game with N players? What can mathematics tell us about how to win a date?

REFERENCES

1. D. André, Solution directe du problème résolu par M. Bertrand, *Comptes Rendus de l'Académie des Sciences* **105** (1887), 136–7.
2. J. Conway, *Functions of One Complex Variable*, 2nd ed., (*Graduate Texts in Mathematics*, vol. 11), Springer-Verlag, New York, 1978.
3. P. Hilton and J. Pedersen, Catalan numbers, their generalizations, and their uses, *Mathematical Intelligencer* **13** (1991), 64–75.
4. S. Stahl, *A Gentle Introduction to Game Theory*, (*Mathematical World*, vol. 13), American Mathematical Society, Providence, 1999.
5. R. Stanley, *Enumerative Combinatorics*, Cambridge University Press, Cambridge, 1999.
6. P. Straffin, *Game Theory and Strategy*, (*New Mathematical Library*, vol. 36), Mathematical Association of America, Washington, 1993.