Quiz 08: Chapter 15

Due: Friday 17 Feb 23

Examine the solved problem below. There are five errors in the solution below. Your task is to locate and identify those errors, then correct them and calculate the proper result. If the same error occurs more than once, only count it as a single error, even if you have to correct it in more than one instance.

Each correctly identified error is worth 3 points, and the re-calculated results are worth 5 points as well. You must save your work in pdf format and submit via the Quiz 08 Assignment in the Chapter 15 folder of the in the Quizzes folder in the Online Classroom in Blackboard. Please do not use any other file format than pdf.

A truck is hauling a log (m = 50kg) out of a ditch using a winch attached to the back of the truck. The winch applies a time-dependent force $T = 400\sqrt{t}$ until t = 4s, when the force reaches a constant value of T = 800N, as shown on the graph. The coefficients of static and kinetic friction between the ground and the log are $\mu_s =$ 0.65 and $\mu_k = 0.40$. Determine how much time will it take for the log to reach a speed of $18\frac{m}{c}$.

A) Construct a complete free body diagram and summarize the forces acting on the log:

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$$\sum_{x} F_x = T - f - (mg) \cos \theta$$
$$\sum_{x} F_y = N - (mg) \sin \theta$$

B) If the winch is switched on at $t_0 = 0$, determine the time t_1 required before the log begins to move.

Since log is at rest, force of friction is static, $f_s = \mu_s N$:

$$\sum F_x = T - f_s - (mg) \cos \theta = 0$$

$$\sum F_y = N - (mg) \sin \theta = 0$$

$$T = (mg) \cos \theta + \mu_s (mg) \sin \theta$$

$$400\sqrt{t} = (mg)[\cos \theta + \mu_s \sin \theta]$$

$$t_1 = \left\{ \frac{(50)(9.8)[\cos 20^\circ + (0.65) \sin 20^\circ]}{400} \right\}^2 = 2.03s$$



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C) Write the impulse-momentum statement for the log and calculate its speed v_2 when $t_2 = 4s$, when the tension reaches its constant value.

Now that the log is in motion, the force of friction is kinetic, $f_k = \mu_k N$:

$$mv_{1} + \int_{t_{1}}^{t_{2}} [T - f_{k} - (mg)\cos\theta]dt = mv_{2}$$

$$0 + \int_{2.03}^{4} \left[400\sqrt{t} - (mg)[\mu_{k}\sin\theta + \cos\theta] \right] dt = (85\text{kg})v_{2}$$

$$\left\{ 400 \left[\frac{3}{2}t^{\frac{3}{2}} \right] - (mg)[\mu_{k}\sin\theta + \cos\theta]t \right\} \Big|_{2.03}^{4} = (85\text{kg})v_{2}$$

$$600 \left(\sqrt{t^{3}}\right) \Big|_{2.03}^{4} - (50\text{kg}) \left(9.8\frac{\text{m}}{\text{s}^{2}} \right) (0.65\sin 20^{\circ} + \cos 20^{\circ})t \Big|_{2.03}^{4} = (50\text{kg})v_{2}$$

$$600\sqrt{(4 - 2.03)^{3}} - (50\text{kg}) \left(9.8\frac{\text{m}}{\text{s}^{2}} \right) (0.65\sin 20^{\circ} + \cos 20^{\circ})(4 - 2.03) = (50\text{kg})v_{2}$$

$$540.0\frac{\text{kg}\cdot\text{m}}{\text{s}} = (50\text{kg})v_{2}$$

$$v_{2} = 10.8\frac{\text{m}}{\text{s}}$$

D) Calculate how much total time it takes for the log to reach $v_3 = 18\frac{\text{m}}{\text{s}}$.

$$\begin{split} mv_2 + [T - f_k - (mg)\cos\theta]t_3 &= mv_3\\ mv_2 + [T - \mu_k(mg)\sin\theta - (mg)\cos\theta]t_3 &= mv_3\\ mv_2 + [800 - (mg)(\mu_k\sin\theta + \cos\theta)]t_3 &= mv_3\\ \Big[800 - (50\text{kg}) \left(9.8\frac{\text{m}}{\text{s}^2}\right) (0.40\sin20^\circ + \cos20^\circ) \Big]t_3 &= (50\text{kg}) \left(18\frac{\text{m}}{\text{s}} - 10.8\frac{\text{m}}{\text{s}}\right)\\ t_3 &= 1.321\text{s}\\ t &= t_1 + 4\text{s} + t_3 = 2.03\text{s} + 4\text{s} + 1.321\text{s} = 7.35\text{s} \end{split}$$