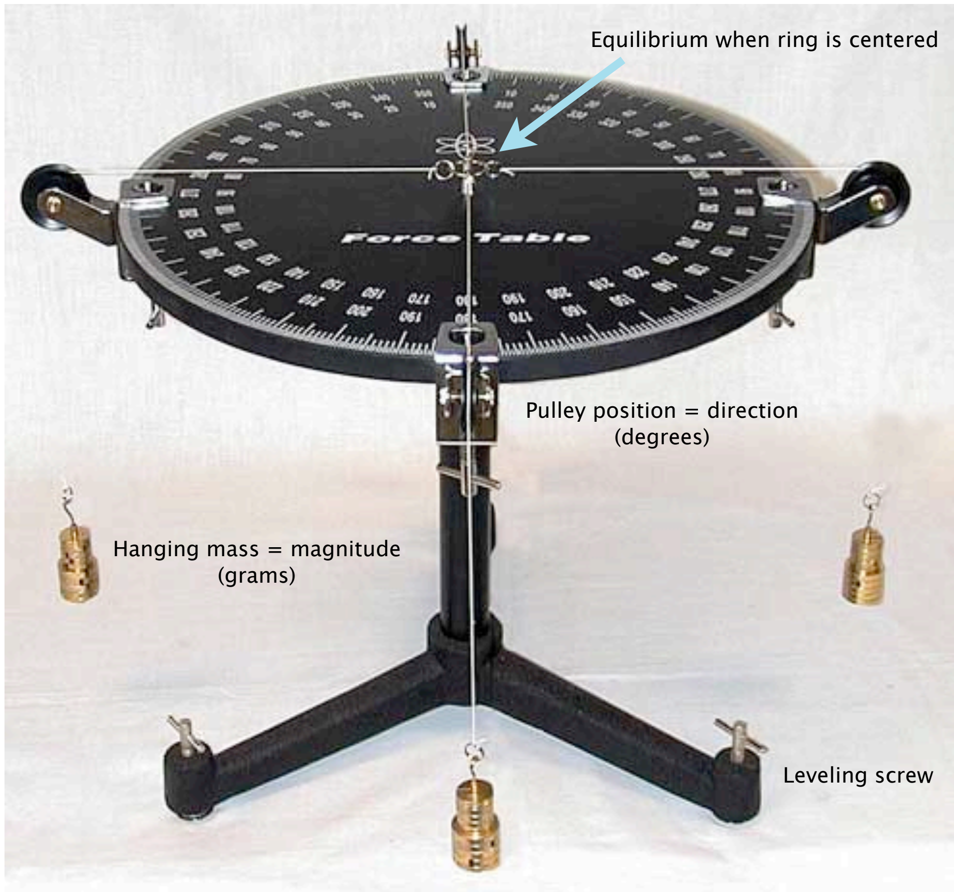
LAB 02: VECTOR ARITHMETIC

The Objectives

Because so many of the physical quantities with which we will be concerned this semester are vectors, it is critically important for us to begin to understand how vectors are different from scalars, and how to mathematically manipulate these vectors. Just like any regular numbers, vectors can be added, subtracted, multiplied, or divided. While we are very comfortable doing these arithmetic operations in general, we need some dedicated practice to get good at doing vector math. Because a vector has both magnitude and direction, the processes are a little different than we are used to. Not difficult, just different.

Initially, we will focus on the methods for adding and subtracting vectors. There are several methods, which is convenient, because it will allow us to check what we have done for correctness. Specifically, we will:

- Construct accurately scaled vector diagrams
- Add vectors using geometric methods
- Resolve two-dimensional vectors into rectangular (cartesian) components
- Use cartesian components to add vectors arithmetically
- Observe the condition for static equilibrium: $\sum \mathbf{F} = 0$



The Procedure

1. Level the force table (if necessary) by adjusting the leveling screws on the feet of the table.
2. The **magnitude** of each vector will be the quantity of mass (in grams) hanging from the string.
3. The direction of each vector is specified by the angle (in degrees) at which the pulley is fixed to the table.
4. The table is balanced ($\sum \mathbf{F} = 0$) when the center ring is centered over the pin at the middle of the table.
5. There are four pulleys. If you are setting up and solving a problem involving fewer than four vectors, you should remove the unnecessary pulley. You must leave all the strings attached to the center rings, however. Coil up any unused string and keep them on the table, but out of the way of the vectors you are working with.

The Data

You will set up and solve several vector addition problems on the table. For each problem, record the following information:

VECTOR	MAGNITUDE (g)	DIRECTION (°)	X—COMPONENT	Y—COMPONENT
A	A	θ_A	A_x	A_y
B	B	θ_B	B_x	B_y
C	C	θ_C	C_x	C_y
R = A + B	R	θ	R_x	R_y

Obviously, if you are adding more than two vectors, you will need additional space on your table. In this example, the vector **C** is the vector you establish experimentally to balance the table after setting vectors **A** and **B** as instructed below. Each problem that you set up will have the form **A** + **B** + **C** = 0, because you are establishing equilibrium in each case. The vector resultant **R**, then, will be the vector having the *same magnitude*, but *opposite direction* as **C**:

$$\vec{R} = -\vec{C}$$

This means that the direction angle associated with vector R must be adjusted from what you read on the force table:

$$\theta = \theta_C + 180°$$

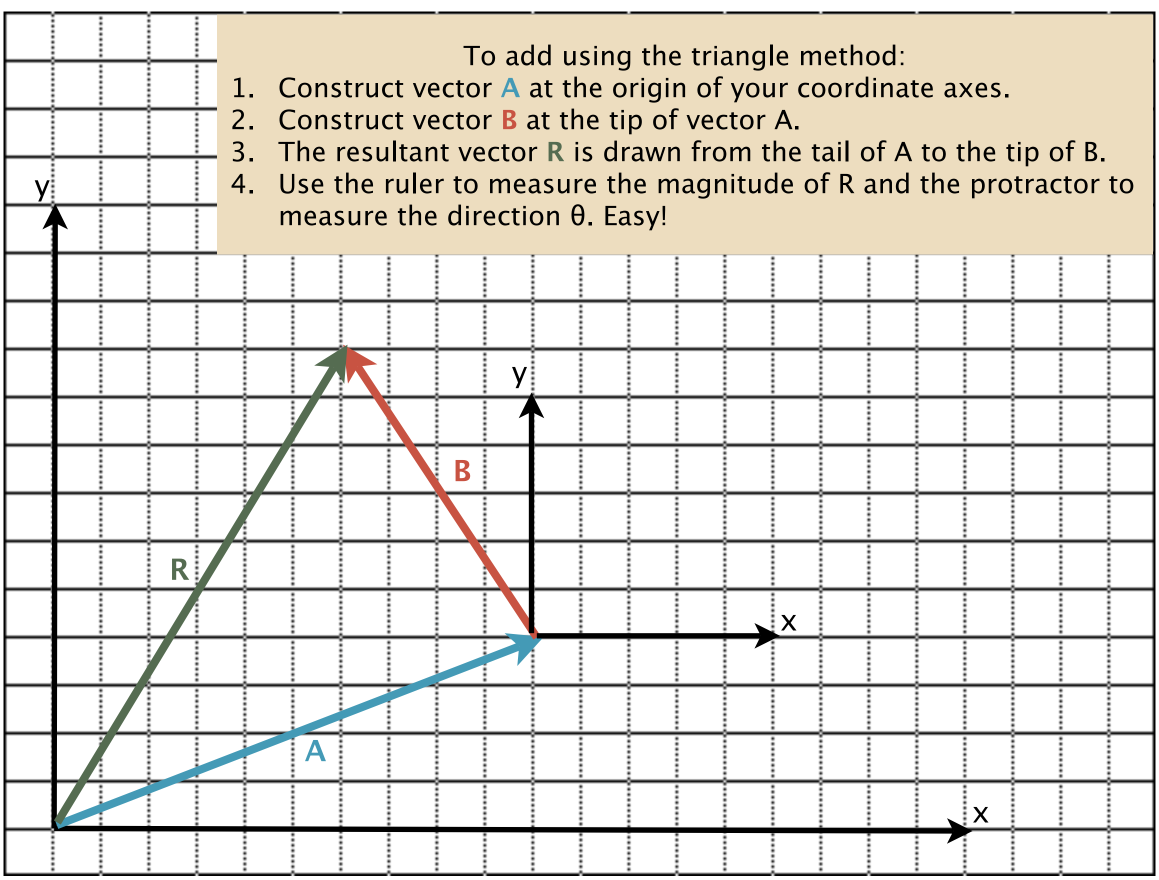
Solve the following vector problems using the force table, then the component and geometric methods:

PROBLEM	VECTOR A	VECTOR B	VECTOR C	RESULTANT
1	100g, 0°	250g, 70°		**R = A + B**
2	150g, 45°	250g, 220°		**R = A + B**
3	200g, 330°	150g, 60°	100g, 135°	**R = A + B + C**
4	250g, 115°	100g, 230°	200g, 315°	**R = A + B + C**

The Data Reduction and Analysis

To resolve vectors **A** and **B** into rectangular components, you must use right-triangle trigonometry. If necessary, you should sketch the triangles in your notebook to insure that you completely understand the idea, and that you are perfectly clear about when to use a sine function and when to use cosine. The actual addition is very simple: keep the x-components together, keep the y-components together. Never mix, never worry. You will also be using the Pythagorean theorem to convert the resultant components R_x and R_y back into a vector expressed as magnitude and direction.

A	B	R	R
$A_x = A\cos\theta_A$	$B_x = B\cos\theta_B$	$R_x = A_x + B_x$	$R = \sqrt{R_x^2 + R_y^2}$
$A_y = A\sin\theta_A$	$B_y = B\sin\theta_B$	$R_y = A_y + B_y$	$\theta = \tan^{-1}\left(\frac{R_y}{R_x}\right)$



Once you have recorded the data from the balanced force table, you should construct an accurately scaled diagram of the problem, and complete a geometric solution. An accurately scaled diagram will permit you to read the solution directly from the paper, using your ruler and protractor, without having to calculate anything. You should consult Section 3.2 of your textbook (pages 73—80) for additional diagrams and example problems to illustrate these methods.

The Conclusions

Compare the results of adding the vectors componentwise to the results from the force table and the geometric results. You probably do not have exactly the same resultant vectors in each case. Why? Where does the uncertainty in this experiment lie?