QUIZ 09: PROBLEM SOLVING (CHAPTERS 07 AND 08)

Solve **five** of the following problems. Each problem is worth **six points**, for a total of 30 points. You may choose which five problems to work on and submit. Please work on separate paper, and identify clearly each problem you solve. You must submit your solutions before you leave.

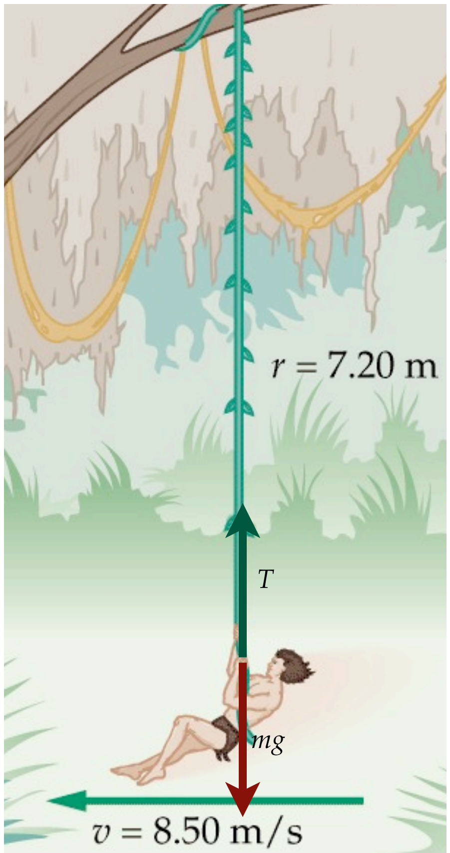
1. On the Wheel of Fortune, a contestant spins the wheel with an initial speed $\boldsymbol{\omega_o = 3.40}$ **rad/s**. After **1.25 revolutions**, the wheel comes to rest ($\boldsymbol{\omega = 0}$) on Bankrupt (of course). The greedy joker had $15,000 and the only letter missing was the "z", to solve the "Fran_ Ferdinand" puzzle in the "Overrated Pop Groups from Scotland" category–he knew the answer, but spun anyway. The next guy spun, did not know the answer, and guessed a "k"; the third woman wanted to buy a vowel, was told there were none left, spun the wheel, then asked for a "u" anyway. So when it came back around to Mr. Greedy, he had to take a $300 spin and solve the puzzle. Find the **angular acceleration** of the wheel, and the **time** for it to come to rest.

$$\omega_o = 3.40\,\tfrac{\text{rad}}{\text{s}}$$
$$\theta = 1.25\text{rev} = 2.5\pi\text{rad}$$
$$\omega = 0$$

$$\omega^2 = \omega_o^2 + 2\alpha\theta$$
$$0 = \left(3.40\,\tfrac{\text{rad}}{\text{s}}\right)^2 + 2\alpha(2.5\pi\text{rad})$$
$$\alpha = -0.736\,\tfrac{\text{rad}}{\text{s}^2}$$

$$\omega = \omega_o + \alpha t$$
$$0 = 3.40\,\tfrac{\text{rad}}{\text{s}} + \left(-0.736\,\tfrac{\text{rad}}{\text{s}^2}\right)t$$
$$t = 4.62\text{s}$$

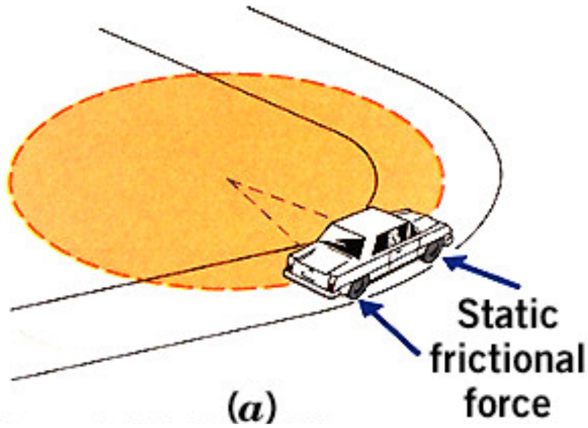
2. Tarzan is up to his old swinging–from–the–vines trick again (Jane is bored, and filing her nails back in the treehouse, listening to Franz Ferdinand on her iPod). His vine is **7.2m long**, and he releases himself from **rest** (there's a neat trick!). At the very **bottom** of his swing (over the dangerously crocodile–infested river) his **tangential speed is 8.5 m/s**. Find his **angular speed**, **centripetal acceleration**, and the **tension** in the vine. He has gained weight since Jane started making him packets of instant pasta alfredo for dinner three nights a week (poor Jane–she is much better at painting her toenails than whipping up a healthy and delicious dinner out of jungle roots and berries; he should trade her in for MaryAnn–at least *she* knows how to make coconut cream pies, which would not be any less fattening, but significantly improves the potential for pie–in–the–face comedy). Anyway, he now masses a robust **120 kg**. The vine will break if the **tension exceeds 1500 N**. Is Tarzan crocodile fodder?



$$v = 8.5\,\tfrac{\text{m}}{\text{s}}$$
$$r = 7.2\text{m}$$
$$\omega = \frac{v}{r} = \frac{8.5\,\frac{\text{m}}{\text{s}}}{7.2\text{m}} = 1.18\,\tfrac{\text{rad}}{\text{s}}$$
$$a_c = \frac{v^2}{r} = \frac{\left(8.5\,\frac{\text{m}}{\text{s}}\right)^2}{7.2\text{m}} = 10.0\,\tfrac{\text{m}}{\text{s}^2}$$
$$T - mg = ma_c$$
$$T = m(g + a_c)$$
$$T = (120\text{kg})(9.8 + 10.0)\tfrac{\text{m}}{\text{s}^2} = 2376\text{N}$$

Since T > 1500N, poor chubby Tarzan breaks the vine, lands in the river, and is tragically eaten by crocodiles.

3. BatManuel has just made the last payment on the superhero supercar of his dreams: a fully restored 1959 Plymouth Fury with the original polyspheric–head V8 and tailfins so sharp they will put your eye out if you look too closely at them. It's black (naturally) with whitewall tires, but he has not had the time (or money–BatManuel is no independently wealthy billionaire philanthropist, he's just a guy who works for the DOT and hangs out at the Superhero's Crime Fighting Club when he's not at his cousin Chuy's garage picking up a few extra bucks detailing lowriders) to install all those neat superhero gadgets that all superhero supercars should have. There are no ejection seats or rocket jets or inflatable pontoons that deploy at the touch of a button. Anyway, he's out driving it because what's the point of owning a fully restored '59 Fury if you can't even enjoy a nice drive around the block? The car drives around a **flat curve at a constant speed**. If the **radius** of the curve is **25m**, and the coefficient of friction between the rubber and the road is **μ_s = 0.85**, how **fast** can he safely take the curve (no slipping)?

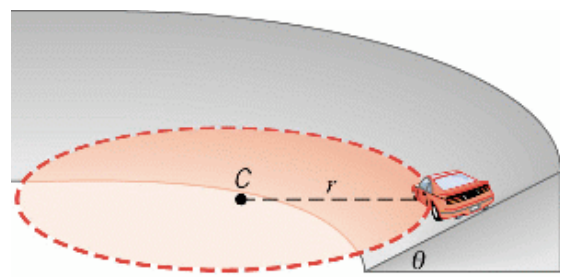


$$\mu_s = 0.85$$
$$r = 25\text{m}$$
$$\sum F_x = f_s = \mu_s N = ma_c$$
$$\sum F_y = N - mg = 0$$

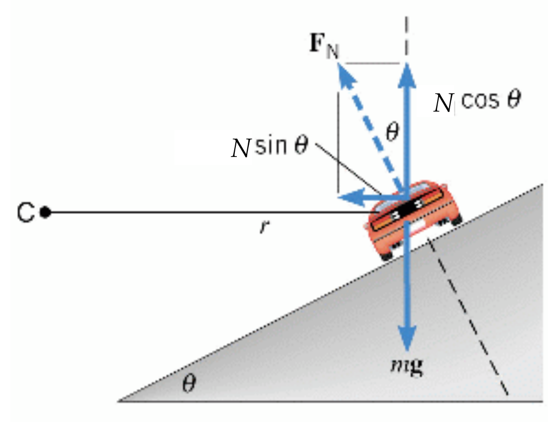
$$N = mg$$
$$\mu_s(mg) = m\left(\frac{v^2}{r}\right)$$
$$v = \sqrt{\mu_s g r}$$
$$v = \sqrt{(0.85)\left(9.8\,\tfrac{\text{m}}{\text{s}^2}\right)(25\text{m})} = 14.4\,\tfrac{\text{m}}{\text{s}}$$

3. What if the curve was banked at **10°**?

$$\sum F_x = N\sin\theta = ma_c$$
$$\sum F_y = N\cos\theta - mg = 0$$
$$N = \frac{mg}{\cos\theta}$$
$$\left(\frac{mg}{\cos\theta}\right)\sin\theta = m\left(\frac{v^2}{r}\right)$$
$$v = \sqrt{gr\tan\theta}$$
$$v = \sqrt{\left(9.8\,\tfrac{\text{m}}{\text{s}^2}\right)(25\text{m})\tan 10°}$$
$$v = 6.57\,\tfrac{\text{m}}{\text{s}}$$



(a)



(b)

If you want to see the solution that includes *both* friction *and* the banked curve, I am happy to show you. But I assumed that everyone would treat this as a banked curve only, since that was how we set it up in class.

4. Speaking of Gilligan's Island, the Professor has just finished making a washing machine, in a desperate attempt to win the heart of MaryAnn (who has recently been having strange an inexplicable dreams about making coconut cream pies in a treehouse for a certain virile jungle–hero). Anyway, Gilligan turns the crank, and the drum of the washing machine spins. It's a top-loading machine, but without that pesky and inefficient agitator–they could wash a queen–sized comforter in this machine, if they had a queen–sized comforter; but they always seemed to have way too much stuff for a simple three–hour tour, so you never know. At any rate, at least everyone's going to have a clean hammock tonight! The drum has a **radius** of **0.30m**, and spins with respect to a central axis that is vertical. The coefficient of friction between the wet hammocks and the drum is **μ_s = 0.50**. If Gilligan can only crank fast enough to spin the drum at **3 revolutions per second**, is he spinning **fast enough** to keep the hammocks stuck to the wall of the drum? Find the **minimum friction coefficient** necessary to keep the hammocks pinned to the drum.

$$\mu_s = 0.50$$

$$r = 0.30\text{m}$$

$$\omega = 3\,\tfrac{\text{rev}}{\text{s}} = 6\pi\,\tfrac{\text{rad}}{\text{s}} = 18.8\,\tfrac{\text{rad}}{\text{s}}$$

$$\sum F_x = N = ma_c$$

$$\sum F_y = f_s - mg = 0$$

$$f_s = \mu_s N = mg$$

$$N = \frac{mg}{\mu_s}$$

$$\left(\frac{mg}{\mu_s}\right) = m\omega^2 r$$

$$\omega_{\min} = \sqrt{\frac{g}{\mu_s r}} = \sqrt{\frac{\left(9.8\,\tfrac{\text{m}}{\text{s}^2}\right)}{(0.50)(0.30\text{m})}} = 8.08\,\tfrac{\text{rad}}{\text{s}}$$

$$\mu_{\min} = \frac{g}{\omega^2 r} = \frac{\left(9.8\,\tfrac{\text{m}}{\text{s}^2}\right)}{\left(18.8\,\tfrac{\text{rad}}{\text{s}}\right)^2(0.30\text{m})} = 0.092$$

Since Gilligan spins faster than the minimum required value, the hammocks remain stuck to the drum.

5. Major Tom (played by Russell Brand in his first starring role! This could be his big break in America!) has taken his protein pills and kept his helmet on, so Ground Control (Ed Harris and Sigourney Weaver, but mostly Ed Harris; the obvious backstory involving their shattered romance is not relevant–yet. But we will see the tension between them and conclude that they really still love each other, and that it will only take one good crisis in the third act (like maybe Major Tom's slow descent into madness, or maybe it's his computer that slowly descends into madness...) to send them tumbling back into each other's arms.) has given him some orbital mechanics to work on. He is on the last leg of his journey, the first manned mission to Mars. Mars has a mass **M = 6.42x10²³kg**, and rotates on its axis with a period of **1.026 standard days**. Major Tom needs to put his spaceship (the *Ziggy Stardust*) in a **geosynchronous orbit** over the planet's equator (to investigate the sudden appearance of this black monolith thing). Find the **orbital altitude** above the surface of Mars. Determine the **tangential velocity** of the *Ziggy*. While you're at it, find the **acceleration due to gravity** at or near the surface (the Martian equivalent of ***g***). The planetary **diameter** of Mars is **6787 km**.

$$G = 6.67\times10^{-11}\,\tfrac{\text{N}\cdot\text{m}^2}{\text{kg}^2}$$

$$M = 6.42\times10^{23}\text{kg}$$

$$R = \frac{d}{2} = \frac{6.787\times10^6\text{m}}{2} = 3.394\times10^6\text{m}$$

$$T = (1.026\text{day})\left(24\,\tfrac{\text{hr}}{\text{day}}\right)\left(3600\,\tfrac{\text{s}}{\text{hr}}\right)$$

$$T = 88{,}646.4\text{s}$$

$$\sum F = G\frac{mM}{r^2} = ma_c$$

$$G\frac{mM}{r^2} = m\omega^2 r$$

$$r^3 = G\frac{M}{\omega^2} = \frac{GM}{\left(\frac{2\pi}{T}\right)^2} = \frac{GMT^2}{4\pi^2}$$

$$r = \sqrt[3]{\frac{\left(6.67\times10^{-11}\,\tfrac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)\left(6.42\times10^{23}\text{kg}\right)(88{,}646.4\text{s})^2}{4\pi^2}}$$

$$r = 2.04\times10^7\text{m}$$

$$r = R + h$$

$$h = r - R = 2.04\times10^7\text{m} - 3.394\times10^6\text{m}$$

$$h = 1.703\times10^7\text{m}$$

$$v = \frac{2\pi r}{T}$$

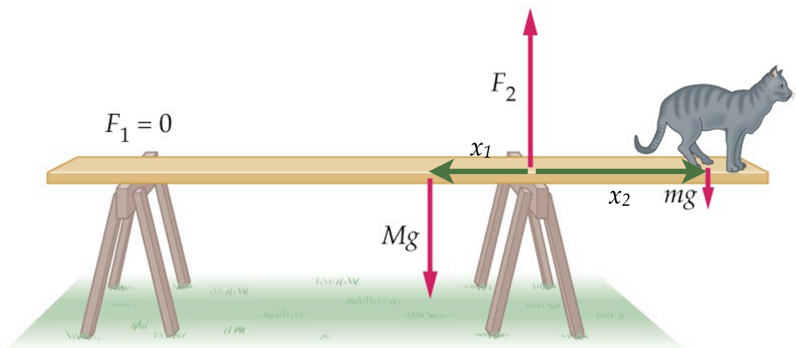
$$v = \frac{2\pi\left(2.04\times10^7\text{m}\right)}{(88646.4\text{s})} = 1448\,\tfrac{\text{m}}{\text{s}}$$

$$G\frac{mM}{R^2} = mg$$

$$g = G\frac{M}{R^2} = \frac{\left(6.67\times10^{-11}\,\tfrac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)\left(6.42\times10^{23}\text{kg}\right)}{\left(6.787\times10^6\text{m}\right)^2}$$

$$g = 3.72\,\tfrac{\text{m}}{\text{s}^2}$$

6. A plank **4m long** is supported by two sawhorses. Why? I have no idea…I'm sorry, but you will have to make up your own story to explain this one. One is **0.5m from the left** end of the plank, and the second is **1.5m from the right** end. The **mass** of the plank is **7kg**. A cat leaps up on the plank. Starting from the center of the plank, he just reaches the **right** end when the plank begins to tip (plank *just* loses contact with the **left** sawhorse). What is the **mass** of the cat?

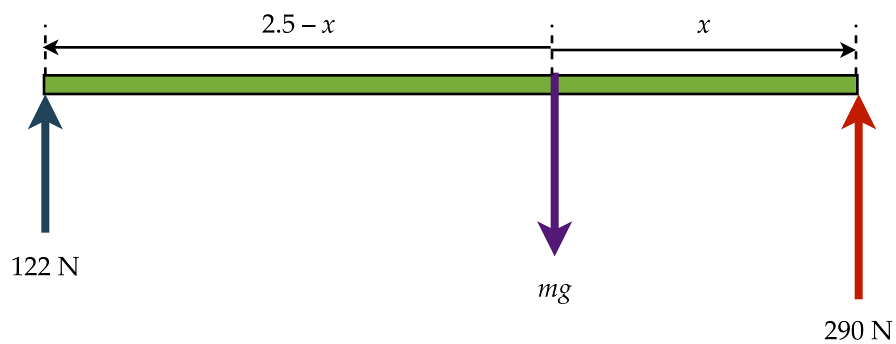


$x_1 = 0.50\text{m} \quad x_2 = 1.5\text{m} \quad M = 7\text{kg}$

$$\sum \tau = (Mg)x_1 - (mg)x_2 = 0$$

$$m = \frac{Mx_1}{x_2} = \frac{(7\text{kg})(0.5\text{m})}{(1.5\text{m})} = 2.3\text{kg}$$

7. Here's one way to find your center of mass: two scales are separated by **2.5m**. A light plank is laid across both, and the scales are re-zeroed (so we can ignore the mass of the plank). You lay down on the plank, with your head directly above one scale. The scale under your head reads **290 N**, and the scale at the other end reads **122 N**. What is your **mass**, and **where** is your **mass center** with respect to your head?



$$\sum F = W_1 + W_2 - mg = 0$$

$$122\text{N} + 290\text{N} = m\left(9.8\,\tfrac{\text{m}}{\text{s}^2}\right)$$

$$m = 42\text{kg}$$

$$\sum \tau_{CM} = W_1(2.5 - x) - W_2 x = 0$$

$$(290\text{N})x - (122\text{N})(2.5 - x) = 0$$

$x = 0.74\text{m}$ from head

$2.5\text{m} - x = 1.76\text{m}$ from other end