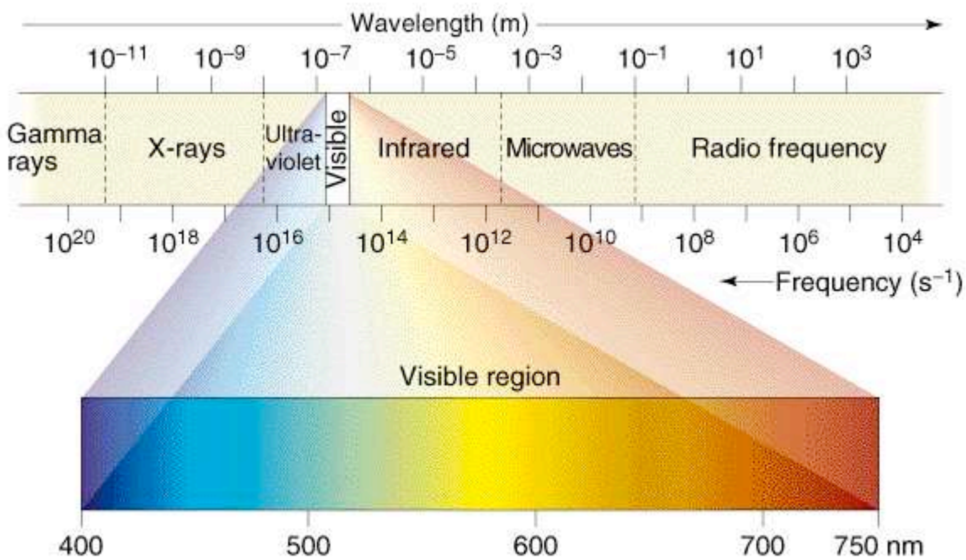
UCA Department of Physics and Astronomy

PHYS 3345: OPTICS

Exam 01: Chapters 02 and 03 — 02/08/08
Part 01: Conceptual Questions



1. What is the difference between a longitudinal and transverse wave?

   Longitudinal: oscillation parallel to propagation
   Transverse: oscillation perpendicular to propagation

2. How do we know that light is a transverse wave?

   Light can be polarized.

3. What is the significance of the condition that **k·r** = constant for a plane wave?

   k·r = constant locates the set of planes perpendicular to the propagation vector, and the value of the constant sets the value of ψ on the plane.

4. What does it mean to state that **E** is linearly polarized in the y-direction?

   E oscillates in the y-direction.

5. What is the physical meaning of the magnitude of the Poynting vector? Why is it necessary to take a time-averaged value?

   S = energy per area per time, or power per area. Irradiance is a power distribution. $\langle S \rangle_T$ because the instantaneous value of S depends on frequency, so it varies rapidly.

6. As an e·m wave travels through space, what carries more energy, the **E** field or the **B** field?

   Neither. They carry the same amount of energy.

7. How do we know that light is a stream of quantized photons?

   Photoelectric effect, Figure 3.20, Figure 3.21.

8. How does the fact that photons are bosons (as opposed to fermions) explain why light appears continuous?

   Bosons can occupy the same state (fermions can't: PEP). Get enough photons together in the same state, light looks continuous.

9. Distinguish between coherent and incoherent light.

   Coherent: monochromatic plane wave. Photon counts follow Poisson distribution. Example: laser.
   Incoherent: polychromatic, chaotic (non-constant irradiance). Bose-Einstein statistics. Example: light bulb.

10. Why doesn't a stationary charge radiate e·m energy?

    If charge is stationary, no **B** field. (Also, **E** is constant, so does not vary with time.)

11. For a linearly accelerating charge, in what direction will the irradiance be greatest?

    Perpendicular to the acceleration.

12. Describe the process by which a ground-state valence electron can make a quantum jump.

    A ground state e^- absorbs the energy of an incoming photon, jumps to a higher energy level. The excited e^- either re-radiates, emitting a photon with equal energy as it falls back to ground state, or the energy is dissipated as heat when the e^- returns to ground.

13. Explain dissipative absorption.

    See above.

14. What is the difference between resonant and non-resonant, or ground state, scattering?

    Resonant scattering: incoming energy at resonant frequency, ground-state electrons excited to higher states.
    Non-resonant scattering: non-resonant frequency, instead of e^- excitation, entire cloud oscillates. Atom is a dipole oscillator.

15. Why does the index of refraction of a medium depend on the frequency of the incident radiation?

    To quote the text (p 68), "the dependence of n on is ω governed by the interplay of the various electric polarization mechanisms contributing at the particular frequency."

    $$n = \sqrt{\frac{\varepsilon}{\varepsilon_o}}$$

    The value of ϵ depends on frequency because an individual atom's polarization is in response to the incoming radiation. As the frequency increases, the atom is less and less able to respond to the changing incident **E** field. Index n increases as ω approaches ω_0, resonant frequency.

Part 02: Problem Solving

Solve each of the following problems **clearly** and **concisely** on separate paper. When necessary, use a sketch to illustrate. Always include the appropriate units on your answers. Plan to spend about 30 minutes answering this set of questions. You may use your textbook as a reference, but no additional reference materials.

Avoid vector confusion by using the unit vector notation shown:

$$\hat{x} = \hat{i} \qquad \hat{y} = \hat{j} \qquad \hat{z} = \hat{k}$$

1. (10 pt) Does the function

$$\psi(z,t) = A\exp\left(Bz^2 + BC^2t^2 - 2BCzt\right)$$

satisfy the wave equation? If so, **show how** and find the wave speed. If not, **show** why not.

$$\psi(z,t) = A\exp\left[B(z-Ct)^2\right]$$

$$\psi(z,t) = f(z-vt)$$

$$v = C$$

If you're feeling spectacularly ambitious, or maybe a little guilty for taking 10 points for three lines of work:

$$\frac{\partial\psi}{\partial z} = 2AB(z-Ct)\exp\left[B(z-Ct)^2\right]$$

$$\frac{\partial\psi}{\partial z} = 2B(z-Ct)\psi(z,t)$$

$$\frac{\partial^2\psi}{\partial z^2} = 2B\psi(z,t) + 2B(z-Ct)\frac{\partial\psi}{\partial z}$$

$$\frac{\partial^2\psi}{\partial z^2} = 2B\psi(z,t) + 4B^2(z-Ct)^2\psi(z,t)$$

$$\frac{\partial^2\psi}{\partial z^2} = 2\left[B + 2B^2(z-Ct)^2\right]\psi(z,t)$$

$$\frac{\partial\psi}{\partial t} = -2ABC(z-Ct)\exp\left[B(z-Ct)^2\right]$$

$$\frac{\partial\psi}{\partial t} = -2BC(z-Ct)\psi(z,t)$$

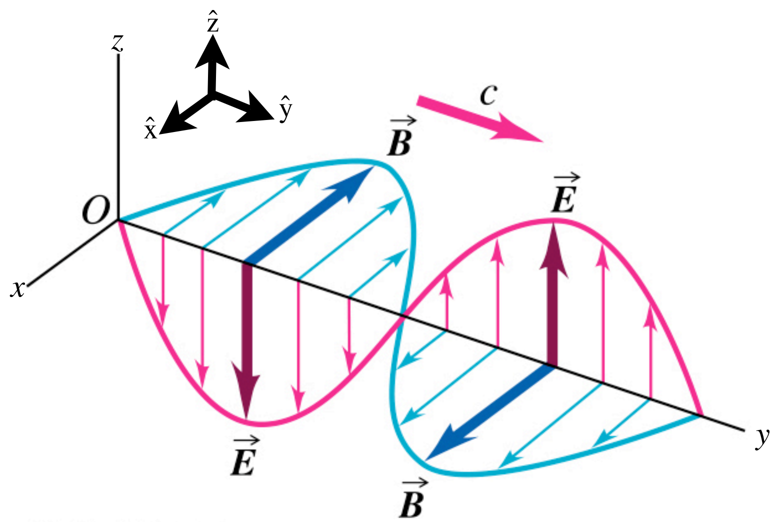
$$\frac{\partial^2\psi}{\partial t^2} = 2BC^2\psi(z,t) - 2BC(z-Ct)\frac{\partial\psi}{\partial t}$$

$$\frac{\partial^2\psi}{\partial t^2} = 2BC^2\psi(z,t) + 4B^2C^2(z-Ct)^2\psi(z,t)$$

$$\frac{\partial^2\psi}{\partial t^2} = 2C^2\left[B + B^2(z-Ct)^2\right]\psi(z,t)$$

$$\frac{\partial^2\psi}{\partial z^2} = \frac{1}{C^2}\frac{\partial^2\psi}{\partial t^2}$$

$$\frac{\partial^2\psi}{\partial z^2} = \frac{1}{v^2}\frac{\partial^2\psi}{\partial t^2}$$



2. The e·m wave shown on the right propagates through free space as shown. If the magnitude of the electric field $\mathbf{E_o}$ = **50V/m**, and the wavelength **λ= 550nm**.

A) (4 pt) Write the propagation vector **k**.

$$\vec{k} = \left(\frac{2\pi}{\lambda}\right)\hat{y}$$

$$\vec{k} = \left(\frac{2\pi}{550\times10^{-9}\,\mathrm{m}}\right)\hat{y}$$

$$\vec{k} = \left(1.14\times10^{7}\,\mathrm{m^{-1}}\right)\hat{y}$$

B) (4 pt) Find the angular frequency ω.

$$\omega = 2\pi\upsilon = \frac{2\pi c}{\lambda}$$

$$\omega = \frac{2\pi\left(3\times10^{8}\,\frac{\mathrm{m}}{\mathrm{s}}\right)}{\left(550\times10^{-9}\,\mathrm{m}\right)} = 3.43\times10^{15}\,\mathrm{s^{-1}}$$

C) (6 pt) Express the vectors for **E** and **B**.

$$\vec{E} = E_o\sin(ky - \omega t + \pi)\hat{z}$$

$$\vec{E} = \left(50\,\tfrac{\mathrm{V}}{\mathrm{m}}\right)\sin\left[\left(1.14\times10^{7}\,\mathrm{m^{-1}}\right)y - \left(3.43\times10^{15}\,\mathrm{s^{-1}}\right)t + \pi\right]\hat{z}$$

$$\vec{B} = B_o\sin(ky - \omega t + \pi)\hat{x}$$

$$\vec{B} = \left(\frac{E_o}{c}\right)\sin(ky - \omega t + \pi)\hat{x}$$

$$\vec{B} = \left(\frac{50\,\frac{\mathrm{V}}{\mathrm{m}}}{3\times10^{8}\,\frac{\mathrm{m}}{\mathrm{s}}}\right)\sin\left[\left(1.14\times10^{7}\,\mathrm{m^{-1}}\right)y - \left(3.43\times10^{15}\,\mathrm{s^{-1}}\right)t + \pi\right]\hat{x}$$

$$\vec{B} = \left(1.67\times10^{-7}\,\mathrm{T}\right)\sin\left[\left(1.14\times10^{7}\,\mathrm{m^{-1}}\right)y - \left(3.43\times10^{15}\,\mathrm{s^{-1}}\right)t + \pi\right]\hat{x}$$

Part 02: Problem Solving

D) (4 pt) Determine the irradiance.

$$I = \frac{c\varepsilon_o}{2}E_o^2$$

$$I = \frac{\left(3\times10^8\ \tfrac{\mathrm{m}}{\mathrm{s}}\right)\left(8.85\times10^{-12}\ \tfrac{\mathrm{C}^2}{\mathrm{N\cdot m}^2}\right)}{2}\left(50\ \tfrac{\mathrm{V}}{\mathrm{m}}\right)^2$$

$$I = 3.32\ \tfrac{\mathrm{W}}{\mathrm{m}^2}$$

3. A different observer perceives the same wave to have a propagation vector in the direction

$$\hat{\mathrm{k}} = \left(\frac{1}{\sqrt{5}}\right)\hat{\mathrm{y}} - \left(\frac{2}{\sqrt{5}}\right)\hat{\mathrm{z}}$$

(meaning his frame of reference, or coordinate system, is different—but nothing else is).

A) (5 pt) Re-write the electric field vector **E** to reflect this change in coordinate axes.

Since **k** is in the yz-plane, **E** will also be in the yz-plane. Because **E** ⊥ **k**, then the direction of E is determined by **E·k** = 0. Thus,

$$\hat{E} = -k_z\hat{\mathrm{y}} + k_y\hat{\mathrm{z}}$$

$$\vec{E}_o = E_o\hat{E}$$

$$\vec{E}_o = \left(50\ \tfrac{\mathrm{V}}{\mathrm{m}}\right)\left[\frac{2}{\sqrt{5}}\hat{\mathrm{y}} + \frac{1}{\sqrt{5}}\hat{\mathrm{z}}\right]$$

$$\vec{E} = \vec{E}_o\sin\left(\vec{k}\cdot\vec{r} - \omega t + \pi\right)$$

$$\vec{k}\cdot\vec{r} = \left(1.14\times10^7\mathrm{m}^{-1}\right)\left[\left(\frac{1}{\sqrt{5}}\right)y - \left(\frac{2}{\sqrt{5}}\right)z\right]$$

$$\vec{k}\cdot\vec{r} = \left(5.11\times10^6\mathrm{m}^{-1}\right)\left(y - 2z\right)$$

$$\vec{E} = \left(50\ \tfrac{\mathrm{V}}{\mathrm{m}}\right)\left[\frac{2}{\sqrt{5}}\hat{\mathrm{y}} + \frac{1}{\sqrt{5}}\hat{\mathrm{z}}\right]\sin\left[\left(5.11\times10^6\mathrm{m}^{-1}\right)\left(y-2z\right) - \left(3.43\times10^{15}\mathrm{s}^{-1}\right)t + \pi\right]$$

B) (2 pt) How does the expression for **B** change?

$\mathbf{B}_o$ doesn't change. The new coordinate system does not affect the direction of the x-axis.

$$\vec{B}_o = \left(\frac{E_o}{c}\right)\hat{\mathrm{x}}$$

$$\vec{B}_o = \left(1.67\times10^{-7}\mathrm{T}\right)\hat{\mathrm{x}}$$

$$\vec{B} = \vec{B}_o\sin\left[\vec{k}\cdot\vec{r} - \omega t + \pi\right]$$

$$\vec{B} = \left(1.67\times10^{-7}\mathrm{T}\right)\sin\left[\left(5.11\times10^6\mathrm{m}^{-1}\right)\left(y-2z\right) - \left(3.43\times10^{15}\mathrm{s}^{-1}\right)t + \pi\right]\hat{\mathrm{x}}$$

4. A laser pointer emits a beam of red light (λ= **700nm**) with diameter **1.00mm**. It's very low power, only **1.00mW**.

A) (4 pt) Calculate the beam irradiance.

$$I = \frac{P}{A} = \frac{P}{\pi r^2}$$

$$I = \frac{\left(1\times10^{-3}\mathrm{W}\right)}{\pi\left(0.5\times10^{-3}\mathrm{m}\right)^2} = 1.27\times10^3\ \tfrac{\mathrm{W}}{\mathrm{m}^2}$$

B) (4 pt) Mean photon flux density?

$$\frac{I}{h\upsilon} = \frac{I\lambda}{hc}$$

$$\frac{I\lambda}{hc} = \frac{\left(1.27\times10^3\ \tfrac{\mathrm{W}}{\mathrm{m}^2}\right)\left(700\times10^{-9}\mathrm{m}\right)}{\left(6.626\times10^{-34}\mathrm{J\cdot s}\right)\left(3\times10^8\ \tfrac{\mathrm{m}}{\mathrm{s}}\right)}$$

$$\frac{I\lambda}{hc} = 4.48\times10^{21}\ \tfrac{\mathrm{photons}}{\mathrm{s\cdot m}^2}$$

C) (4 pt) Mean photon flux?

$$\Phi = \frac{P}{h\upsilon} = \frac{P\lambda}{hc}$$

$$\Phi = \frac{\left(1\times10^{-3}\mathrm{W}\right)\left(700\times10^{-9}\mathrm{m}\right)}{\left(6.626\times10^{-34}\mathrm{J\cdot s}\right)\left(3\times10^8\ \tfrac{\mathrm{m}}{\mathrm{s}}\right)}$$

$$\Phi = 3.52\times10^{15}\ \tfrac{\mathrm{photons}}{\mathrm{s}}$$

D) (4 pt) How much energy does a single photon in the beam carry?

$$E = h\upsilon = \frac{hc}{\lambda}$$

$$E = \frac{\left(6.626\times10^{-34}\mathrm{J\cdot s}\right)\left(3\times10^8\ \tfrac{\mathrm{m}}{\mathrm{s}}\right)}{\left(700\times10^{-9}\mathrm{m}\right)}$$

$$E = 2.84\times10^{-19}\mathrm{J}$$

E) (4 pt) What pressure is exerted on a screen if the beam strikes, and is perfectly reflected?

$$\wp = 2\frac{I}{c} = 2\frac{\left(1.27\times10^3\ \tfrac{\mathrm{W}}{\mathrm{m}^2}\right)}{\left(3\times10^8\ \tfrac{\mathrm{m}}{\mathrm{s}}\right)}$$

$$\wp = 8.49\times10^{-6}\ \tfrac{\mathrm{N}}{\mathrm{m}^2}$$