UCA Department of Physics and Astronomy

PHYS 3345: OPTICS

Assignment 01: Chapter 02 DUE: January 18, 2008

Spring 2008

Work each problem neatly and completely. Unless otherwise noted, each problem is worth **5 points**. You should solve on green engineering paper or blank unruled paper. You must include sufficient demonstration of your problem solving process. If a problem is to be solved by inspection, state this. If graphs or plots are required, you should use an appropriate tool for their construction (there are several options available on the computers in LSC 114). Most of these initial problems are pretty easy; you should expect the level of mathematical rigor to increase sharply, and in short order. I would also take this opportunity to scoop up as many points as possible before it actually gets hard.

1. Hecht, Problem 2.2
   The speed of light in vacuum is $3\text{x}10^8$m/s. Find the wavelength of red light having a frequency of $5\text{x}10^{14}$Hz. Compare this with the wavelength of a 60Hz wave.

$$\lambda_1 = \frac{c}{\upsilon_1} = \frac{3\times10^8\,\frac{\text{m}}{\text{s}}}{5\times10^{14}\,\text{Hz}} = 6\times10^{-7}\,\text{m}$$

$$\lambda_2 = \frac{c}{\upsilon_2} = \frac{3\times10^8\,\frac{\text{m}}{\text{s}}}{60\text{Hz}} = 5\times10^{6}\,\text{m}$$

$$\frac{\lambda_2}{\lambda_1} = \frac{\upsilon_1}{\upsilon_2} = \frac{5\times10^6\,\text{m}}{6\times10^{-7}\,\text{m}} = 8.3\times10^{12}$$

2. Hecht, Problem 2.5
   A vibrating hammer strikes the end of a long metal rod in such a way that a periodic compression wave with a wavelength of 4.3m travels down the rod's length at a speed of 3.5km/s. What is the frequency?

$$\upsilon = \frac{v}{\lambda} = \frac{3500\,\frac{\text{m}}{\text{s}}}{4.3\text{m}} = 814\text{Hz}$$

3. A one-dimensional wave is specified by:

$$y = 15\sin 2\pi\left(4t - 5x + \frac{2}{3}\right)$$

All units are mks. Determine this wave's

a) amplitude
b) wavelength
c) frequency
d) phase speed
e) initial phase

$$y = A\sin(kx - \omega t + \varepsilon)$$

$$y = 15\sin\left(8\pi t - 10\pi x + \frac{4\pi}{3}\right)$$

$$A = 15\text{m}$$

$$k = 10\pi\text{m}^{-1}$$

$$\omega = 8\pi\text{Hz}$$

$$\varepsilon = \frac{4\pi}{3}\text{rad}$$

$$\lambda = \frac{2\pi}{k} = \frac{2\pi}{10\pi} = 0.2\text{m}$$

$$\upsilon = \frac{\omega}{2\pi} = \frac{8\pi}{2\pi} = 4\text{Hz}$$

$$\tau = \frac{1}{\upsilon} = \frac{1}{4\text{Hz}} = 0.25\text{s}$$

There was a substantial amount of re-writing the function so that the argument looked like (kx – ωt), and it was not always executed successfully. Not necessary, and less messing around makes fewer chances to make a mistake.

4. Show that the one-dimensional pulse

$$\psi(x,t) = Ae^{-4\left(x-\frac{3t}{2}\right)^2}$$

satisfies the one-dimensional wave equation.

$$\psi(x,t) = Ae^{-4\left(x-\frac{3t}{2}\right)^2}$$

$$\frac{\partial\psi}{\partial x} = Ae^{-4\left(x-\frac{3t}{2}\right)^2}\left[-4(2)\left(x-\frac{3t}{2}\right)\right] = -8\left(x-\frac{3t}{2}\right)\psi$$

$$\frac{\partial^2\psi}{\partial x^2} = \frac{\partial}{\partial x}\left[-8\left(x-\frac{3t}{2}\right)\psi\right] = -8\psi - 8\left(x-\frac{3t}{2}\right)\frac{\partial\psi}{\partial x}$$

$$\frac{\partial^2\psi}{\partial x^2} = -8\psi + 64\left(x-\frac{3t}{2}\right)^2\psi = 8\psi\left[8\left(x-\frac{3t}{2}\right)^2 - 1\right]$$

$$\frac{\partial\psi}{\partial t} = Ae^{-4\left(x-\frac{3t}{2}\right)^2}\left[-4(2)\left(-\frac{3}{2}\right)\left(x-\frac{3t}{2}\right)\right] = 12\left(x-\frac{3t}{2}\right)\psi$$

$$\frac{\partial^2\psi}{\partial t^2} = \frac{\partial}{\partial t}\left[12\left(x-\frac{3t}{2}\right)\psi\right] = -18\psi + 12\left(x-\frac{3t}{2}\right)\frac{\partial\psi}{\partial t}$$

$$\frac{\partial^2\psi}{\partial t^2} = -18\psi + 144\left(x-\frac{3t}{2}\right)^2\psi = 18\psi\left[8\left(x-\frac{3t}{2}\right)^2 - 1\right]$$

$$\frac{\partial^2\psi}{\partial x^2} = \frac{8}{18}\frac{\partial^2\psi}{\partial t^2}$$

$$\frac{1}{v^2} = \frac{8}{18}$$

$$v^2 = \frac{9}{4}$$

$$v = \frac{3}{2}$$

5. Hecht, Problem 2.22
Write the expression for the wave function of a harmonic wave with amplitude 10^3V/m, period $2.2\text{x}10^{-15}$s, and speed $3\text{x}10^8$m/s. The wave propagates in the negative x-direction and has a value of 10^3V/m at t = 0 and x = 0.

$$A = 10^3\ \tfrac{\text{V}}{\text{m}}$$

$$\tau = 2.2\times10^{-15}\,\text{s}$$

$$v = c = 3\times10^8\ \tfrac{\text{m}}{\text{s}}$$

$$k = \frac{2\pi}{\lambda} = \frac{2\pi\tau}{c} = \frac{2\pi\left(2.2\times10^{-15}\,\text{s}\right)}{3\times10^8\ \tfrac{\text{m}}{\text{s}}} = 9.5\times10^6\,\text{m}^{-1}$$

$$\omega = \frac{2\pi}{\tau} = \frac{2\pi}{2.2\times10^{-15}\,\text{s}} = 2.9\times10^{15}\,\text{s}^{-1}$$

$$\psi(x,t) = A\sin(kx + \omega t + \varepsilon)$$

$$\psi(x,t) = \left(10^3\ \tfrac{\text{V}}{\text{m}}\right)\sin\left[\left(9.5\times10^6\,\text{m}^{-1}\right)x + \left(2.9\times10^{15}\,\text{s}^{-1}\right)t + \varepsilon\right]$$

$$\psi(0,0) = \left(10^3\ \tfrac{\text{V}}{\text{m}}\right)\sin[\varepsilon]$$

$$\sin\varepsilon = 1$$

$$\varepsilon = \frac{\pi}{2}$$

$$\psi(x,t) = \left(10^3\ \tfrac{\text{V}}{\text{m}}\right)\sin\left[\left(9.5\times10^6\,\text{m}^{-1}\right)x + \left(2.9\times10^{15}\,\text{s}^{-1}\right)t + \frac{\pi}{2}\right]$$

6. Hecht, Problem 2.31

(a) $\psi(z, t) = (az - bt)^2$

$$\psi(z,t) = (az - bt)^2$$

$$\psi(z,t) = a^2\left(z - \frac{b}{a}t\right)^2$$

$$\frac{\partial\psi}{\partial z} = 2a^2\left(z - \frac{b}{a}t\right)$$

$$\frac{\partial^2\psi}{\partial z^2} = 2a^2$$

$$\frac{\partial\psi}{\partial t} = 2a^2\left(z - \frac{b}{a}t\right)\left(-\frac{b}{a}\right) = -2ab\left(z - \frac{b}{a}t\right)$$

$$\frac{\partial^2\psi}{\partial t^2} = -2ab\left(-\frac{b}{a}\right) = 2b^2$$

$$\frac{\partial^2\psi}{\partial z^2} = \frac{a^2}{b^2}\frac{\partial^2\psi}{\partial t^2}$$

$$\frac{1}{v^2} = \frac{a^2}{b^2}$$

$$v = \frac{b}{a}$$

(b) $\psi(x, t) = (ax + bt + c)^2$

$$\psi(x,t) = (ax + bt + c)^2$$

$$\psi(x,t) = a^2\left(x + \frac{b}{a}t + \frac{c}{a}\right)^2$$

$$\frac{\partial\psi}{\partial x} = 2a^2\left(x + \frac{b}{a}t + \frac{c}{a}\right)$$

$$\frac{\partial^2\psi}{\partial x^2} = 2a^2$$

$$\frac{\partial\psi}{\partial t} = 2a^2\left(x + \frac{b}{a}t + \frac{c}{a}\right)\left(\frac{b}{a}\right) = 2ab\left(x + \frac{b}{a}t + \frac{c}{a}\right)$$

$$\frac{\partial^2\psi}{\partial t^2} = 2ab\left(\frac{b}{a}\right) = 2b^2$$

$$\frac{\partial^2\psi}{\partial x^2} = \frac{a^2}{b^2}\frac{\partial^2\psi}{\partial t^2}$$

$$\frac{1}{v^2} = \frac{a^2}{b^2}$$

$$v = \frac{b}{a}$$

(c) $\psi(x, t) = 1/(ax^2 + b)$
This can't be a traveling wave since it's not a function of time. No further inspection necessary.

7. Hecht, Problem 2.32

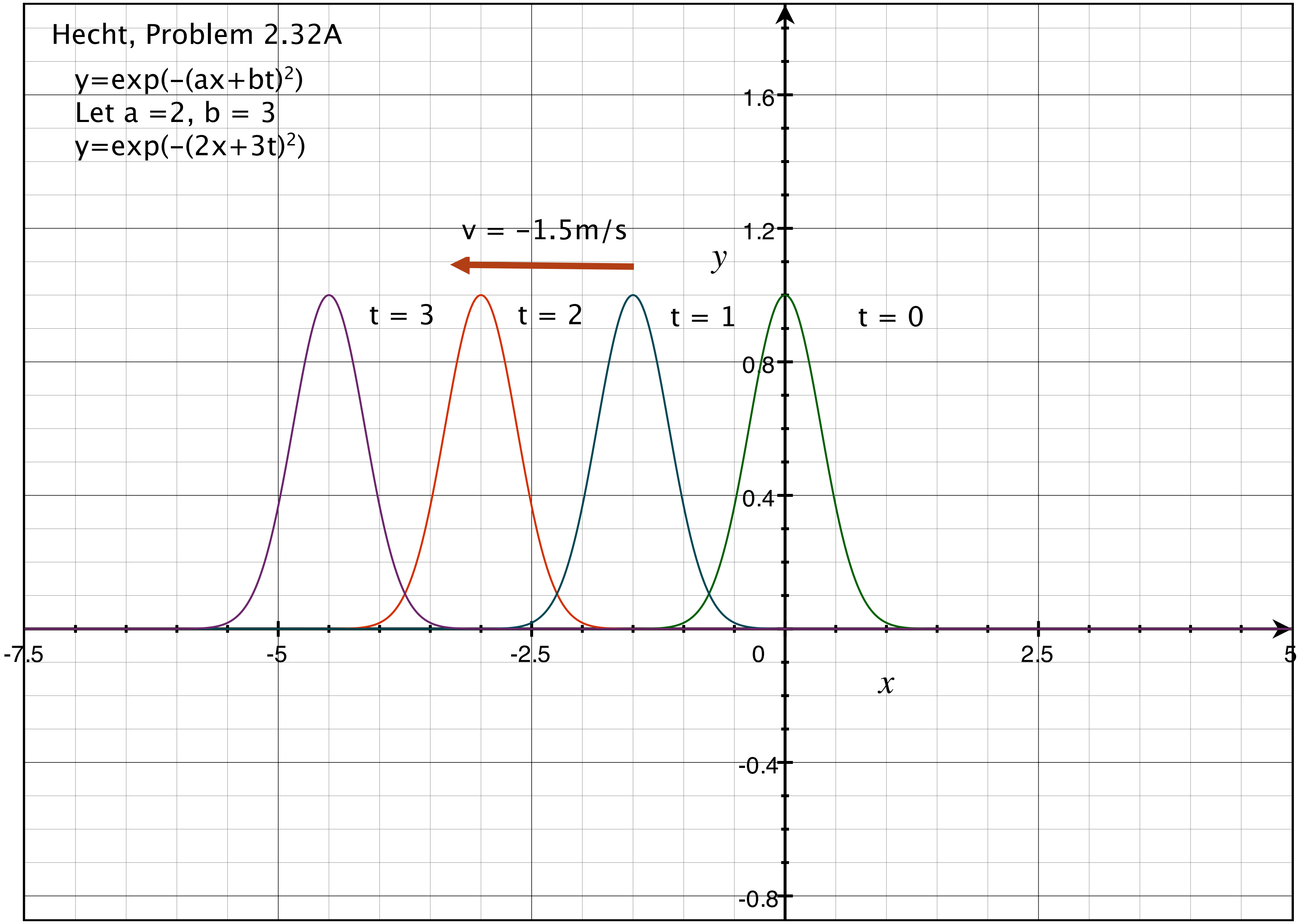
(a) $\psi(y, t) = \exp[-(a^2y^2 + b^2t^2 + 2abty)]$

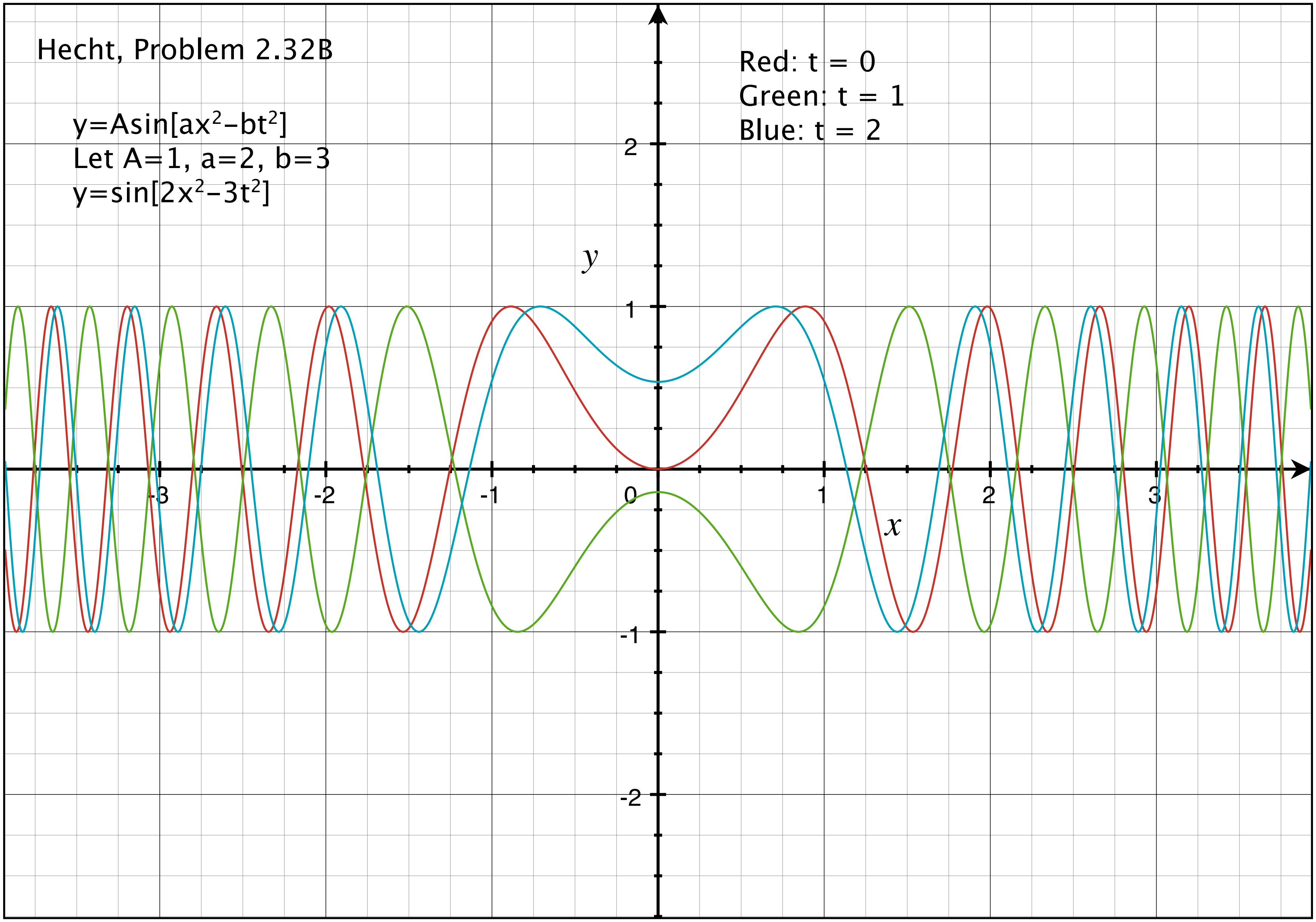
(b) $\psi(z, t) = A\sin(az^2 - bt^2)$

(c) $\psi(x, t) = A\sin[2\pi(x/a + b/t)^2]$

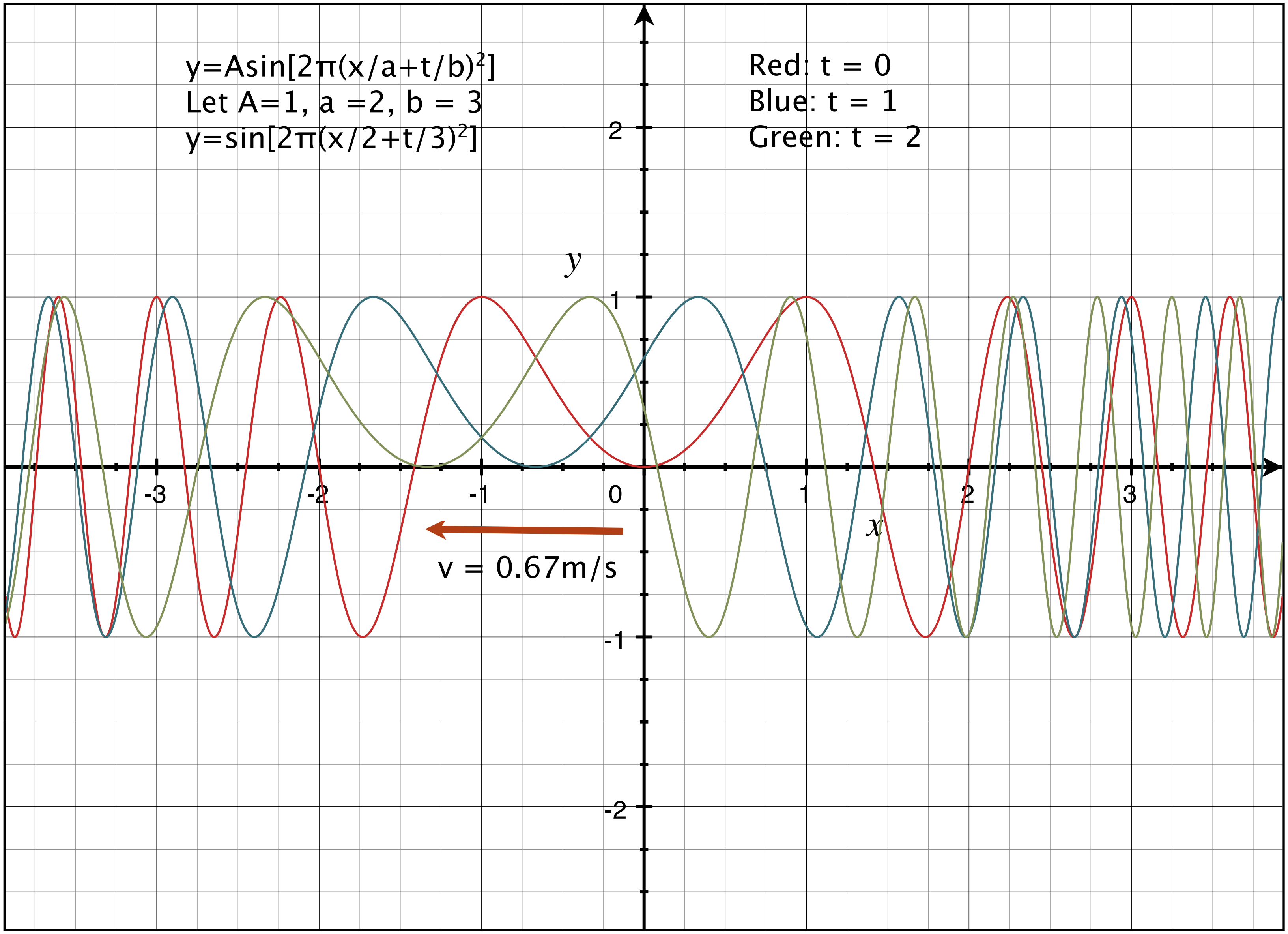
(d) $\psi(x, t) = A\cos^2[2\pi(t - x)]$

Each of these is examined graphically on the following pages. If you need to see the partials that prove the wave equation, I'll show you my paper, but otherwise the graphs make it obvious.


Hecht, Problem 2.32A
y=exp(−(ax+bt)2)
Let a =2, b = 3
y=exp(−(2x+3t)2)
v = −1.5m/s
t = 3
t = 2
t = 1
t = 0
y
x
1.6
1.2
0.8
0.4
-0.4
-0.8
-7.5
-5
-2.5
0
2.5
5


Hecht, Problem 2.32B
y=Asin[ax²–bt²]
Let A=1, a=2, b=3
y=sin[2x²–3t²]
Red: t = 0
Green: t = 1
Blue: t = 2
y
x
2
1
-1
-2
-3
-2
-1
0
1
2
3

Hecht, Problem 2.32C



Hecht, Problem 2.32D