UCA Department of Physics and Astronomy

PHYS 3345: OPTICS

Assignment 08: Chapter 07 Spring 2008

DUE: March 31, 2008

1. Hecht, Problem 7.13

$$\lambda = \frac{c}{v}$$

$$\lambda = \frac{\left(3\times 10^8 \frac{\text{m}}{\text{s}}\right)}{\left(10^{10}\text{Hz}\right)} = 0.03\text{m}$$

Nodes at $\frac{n\lambda}{2}$, or $(0.015\text{m})n$, where n = 0, 1, 2, ...

Anti-nodes at $\frac{(2n+1)\lambda}{4}$, where n = 0, 1, 2, ...

2. Hecht, Problem 7.14

$$E(x,t) = 2E_o \sin(kx)\cos(\omega t)$$

$$E(x,t) = 100\sin\left(\frac{2\pi}{3}x\right)\cos(5\pi t)$$

$$E(x,t) = E_I + E_R$$

$$E(x,t) = E_o\left[\sin(kx+\omega t) + \sin(kx-\omega t)\right]$$

$$E_I = E_o \sin(kx+\omega t)$$

$$E_I = 100\sin\left(\frac{2\pi}{3}x + 5\pi t\right)$$

$$E_R = E_o \sin(kx-\omega t)$$

$$E_R = 100\sin\left(\frac{2\pi}{3}x - 5\pi t\right)$$

3. Hecht, Problem 7.35

$$A_o = \frac{2}{\lambda}\int_0^\lambda f(x)dx$$

$$A_o = \frac{2}{2\pi}\int_0^{2\pi} f(\theta)d\theta$$

$$A_o = \frac{1}{\pi}\int_0^{2\pi} \theta^2 d\theta$$

$$A_o = \frac{1}{\pi}\left(\frac{\theta^3}{3}\right)\Bigg|_0^{2\pi} = \frac{8\pi^2}{3}$$

$$A_m = \frac{2}{\lambda}\int_0^\lambda f(x)\cos(mkx)dx$$

$$A_m = \frac{2}{2\pi}\int_0^{2\pi} f(\theta)\cos(m\theta)d\theta$$

$$A_m = \frac{1}{\pi}\int_0^{2\pi} \theta^2\cos(m\theta)d\theta$$

$$A_m = \frac{1}{\pi}\left[\frac{2\theta\cos(m\theta)}{m^2} + \frac{(m^2\theta^2-2)\sin(m\theta)}{m^3}\right]\Bigg|_0^{2\pi}$$

$$A_m = \frac{4}{m^2}$$

$$B_m = \frac{2}{\lambda}\int_0^\lambda f(x)\sin(mkx)dx$$

$$B_m = \frac{2}{2\pi}\int_0^{2\pi} f(\theta)\sin(m\theta)d\theta$$

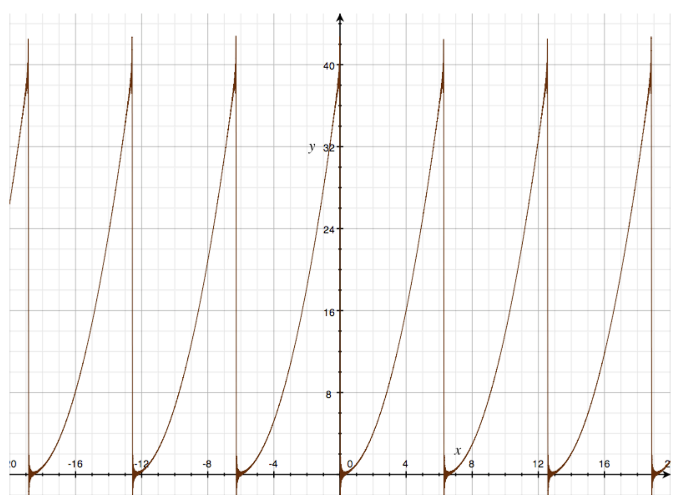
$$B_m = \frac{1}{\pi}\int_0^{2\pi} \theta^2\sin(m\theta)d\theta$$

$$B_m = \frac{1}{\pi}\left[\frac{2\theta\sin(m\theta)}{m^2} - \frac{(m^2\theta^2-2)\sin(m\theta)}{m^3}\right]\Bigg|_0^{2\pi}$$

$$B_m = -\frac{4\pi}{m}$$

$$f(\theta) = \frac{A_o}{2} + \sum_{m=1}^{\infty} A_m\cos(m\theta) + \sum_{m=1}^{\infty} B_m\sin(m\theta)$$

$$f(\theta) = \frac{4\pi^2}{3} + \sum_{m=1}^{\infty}\left[\frac{4}{m^2}\cos(m\theta) - \frac{4\pi}{m}\sin(m\theta)\right]$$



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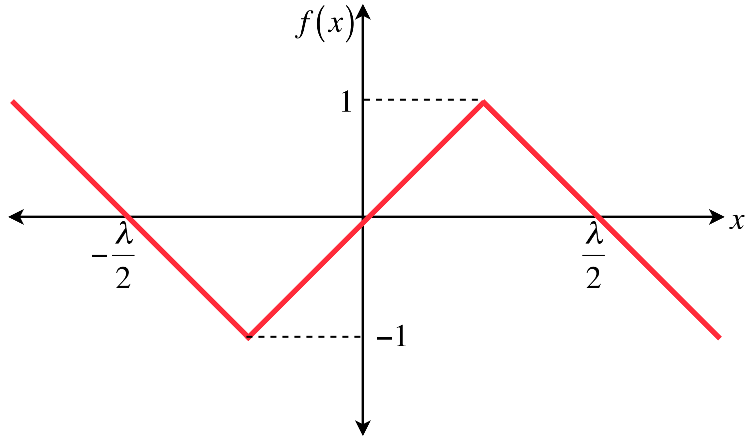
4. Find the Fourier series f(x) for the sawtooth wave shown in the figure below:

$$-\frac{\lambda}{4} < x < \frac{\lambda}{4}$$
$$f(x) = \frac{4}{\lambda}x$$
$$\frac{\lambda}{4} < x < \frac{3\lambda}{4}$$
$$f(x) = 2 - \frac{4}{\lambda}x$$



$$A_m = 0$$
$$B_m = \frac{2}{\lambda}\int_0^{\lambda} f(x)\sin(mkx)\,dx$$

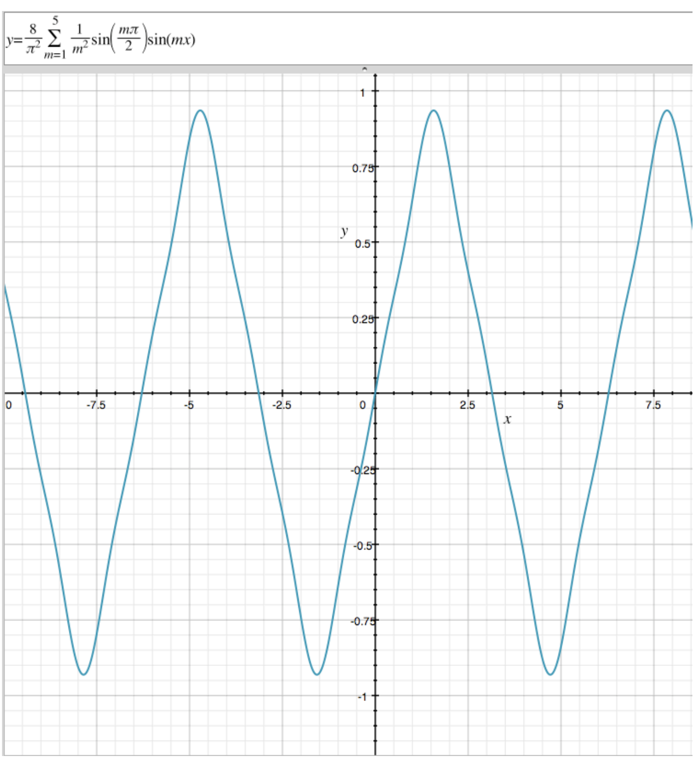
$$B_m = \frac{2}{\lambda}\int_{-\frac{\lambda}{4}}^{\frac{\lambda}{4}}\left(\frac{4}{\lambda}x\right)\sin(mkx)\,dx + \frac{2}{\lambda}\int_{\frac{\lambda}{4}}^{\frac{3\lambda}{4}}\left(2-\frac{4}{\lambda}x\right)\sin(mkx)\,dx$$

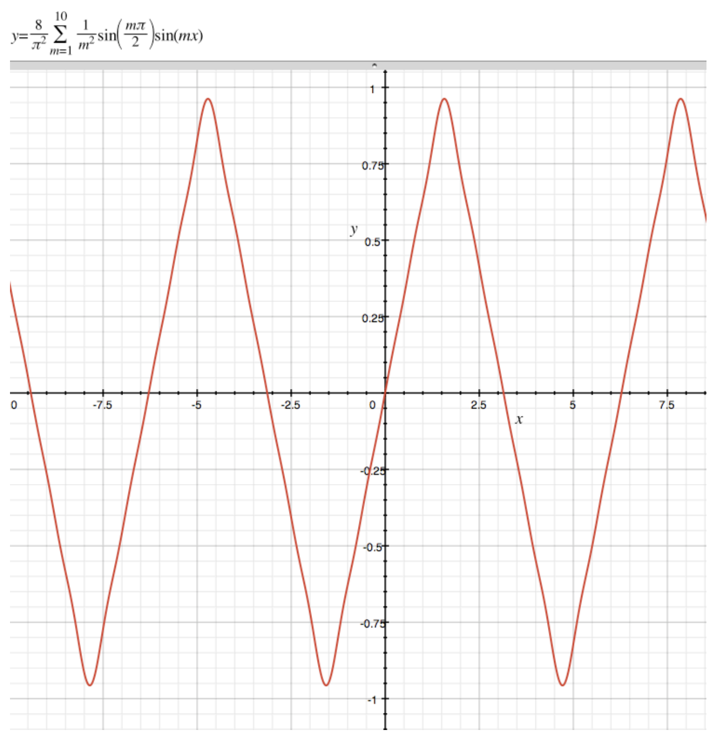
$$B_m = \frac{8}{\lambda^2}\int_{-\frac{\lambda}{4}}^{\frac{\lambda}{4}} x\sin(mkx)\,dx + \frac{4}{\lambda}\int_{\frac{\lambda}{4}}^{\frac{3\lambda}{4}}\left(1-\frac{k}{\pi}x\right)\sin(mkx)\,dx$$

$$B_m = \frac{8}{\lambda^2}\left[\frac{\sin(mkx)-(mkx)\cos(mkx)}{m^2k^2}\right]_{-\frac{\lambda}{4}}^{\frac{\lambda}{4}} + \frac{4}{\lambda}\left[\frac{(kx-\pi)\cos(mkx)}{\pi mk} - \frac{\sin(mkx)}{\pi m^2 k}\right]_{\frac{\lambda}{4}}^{\frac{3\lambda}{4}}$$

$$B_m = \frac{8}{m^2\pi^2}\sin\left(\frac{m\pi}{2}\right)$$
$$f(x) = \sum_{m=1}^{\infty} B_m \sin(mkx)$$
$$f(x) = \frac{8}{\pi^2}\sum_{m=1}^{\infty}\frac{1}{m^2}\sin\left(\frac{m\pi}{2}\right)\sin(mkx)$$

5. Plot the series above using 5 terms, and again using 10. Do you get a reasonable approximation of the sawtooth?





Five terms (left) is still a little raw; 10 terms is better, but notice that neither quite gets the amplitude of the sawtooth up to 1. Graphed with the assumption that k =1, so wavelength = 2π.