Thermal Physics
Test 2
Dr. Addison
March 30, 2001
Due Friday April 6

Answer any 5 questions

1. Consider a dielectric crystal made up of layers of atoms, with rigid coupling between layers so that the motion of the atoms is restricted to the plane of the layer. Show that the phonon heat capacity in the Debye approximation in the low temperature limit is proportional to T^2.

2. According to the Rayliegh-Jeans law, valid for low frequencies, the mean energy per photon (or if you like, the mean energy per normal mode) of blackbody radiation at temperature T is
$$E = kT.$$
According to Wien's law valid at high frequencies,
$$E = \hbar\omega \mathrm{e}^{-\beta\hbar\omega}.$$
Calculate $\mathrm{d}(1/T)/\mathrm{d}E$ for these two cases. Then invent an interpolation formula for $\mathrm{d}(1/T)/\mathrm{d}E$ between these points. By integrating this interpolation formula, deduce an expression for E valid for any frequency. If your interpolation formula is correct, and you evaluate the integration constant by requiring $E \to \infty$ as $T \to \infty$ you will get Planck's law. Carry this out. This is the way that Planck derived his law originally!

3. The energy density (E/V) of blackbody radiation can be written as
$$\frac{E}{V} = \frac{4}{c}\sigma T^4,$$
where σ is a constant and c is the speed of light. Using
$$\left(\frac{\partial S}{\partial E}\right)_{V,N} = \frac{1}{T}$$
show that $S \propto T^3 V$ for a photon gas.

4. Consider a gas of photons in thermal equilibrium in a cube of volume V at temperature T. Let the cavity expand *isentropically* – the radiation pressure will perform work and the temperature will drop. Find the work done by the photons during the expansion. By considering the cosmic 2.7 K radiation as a photon gas, what was the radius of the the Universe when the cosmic radiation was 3000 K compared to its present value. (A ratio will be fine)

5. The velocity of longitudinal sound waves in liquid ^{4}He below 0.6 K is 2.383×10^4 cm s^{-1}. There are no transverse sound waves in the liquid. The density is 0.145 g cm^{-3}. Calculate the Debye temperature. Calculate the heat capacity per gram on the Debye theory and compare with the experimental value $C_v = 0.0204 \times T^3$, in Jg^{-1}K^{-1}. (Note, I have expressed the experimental value per gram of liquid.)

6. Problem 5 on page 142 of your textbook.
7. Problem 18 on page 146 of your textbook.
8. Problem 20 on page 146 of your textbook.