AM466: Finite Element Method

Homework 2
Due in class on Feb 15, Tuesday

1. (10 marks) Do exercise S10 on page 77.

2. (10 marks) Do exercise S11 on page 77.

3. (10 marks) Do exercise N1 on page 78.

4. (20 marks) Do exercise N4 on page 79.

5. Consider the Neumann problem

$$
\begin{aligned}
-y'' &= f(x), \quad 0 < x < 1, \\
y'(0) &= 1, \quad y'(1) = 2.
\end{aligned}
$$

   (a) (15 marks) Develop the linear system of equations describing an approximation of this problem using only two finite elements and linear shape function. Show the stiffness matrix is singular.

   (b) (5 marks) Show that $f(x)$ must satisfy

$$
\int_0^1 f(x)dx = -1.
$$

   (c) (5 marks) By specifying the value y_j of y_h at any node j equal to an arbitrary constant, reduce this system to an invertible 2×2 system of equation and an auxiliary equation for the value of y_h at the specified node.

   (d) (5 marks) Show that the 2×2 system and the auxiliary equation derived above are compatible, that is, yield the same answer.