AM466: Finite Element Method

Homework 4
Due in class on March 15, Tuesday

1. (10 marks) Do exercise S3 on page 89.
2. (10 marks) Do exercise S5 on page 89.
3. (10 marks) Do exercise S3 on page 109.
4. (10 marks) Do exercise S8 on page 110.
5. Consider the Poisson's equation

$$\begin{aligned}
\frac{\partial}{\partial x}\left((1+x^2)\frac{\partial \Phi}{\partial x}\right) + \frac{\partial}{\partial y}\left((1+y^2)\frac{\partial \Phi}{\partial y}\right) - 4xy &= 0, \quad \text{in} \quad \Omega, \\
(1+x^2)\frac{\partial \Phi}{\partial x}n_x + (1+y^2)\frac{\partial \Phi}{\partial y}n_y &= -y, \quad x = 0, \\
(1+x^2)\frac{\partial \Phi}{\partial x}n_x + (1+y^2)\frac{\partial \Phi}{\partial y}n_y &= 65y, \quad x = 8, \\
\Phi &= 0, \quad y = 0, \\
\Phi &= 4y, \quad y = 4.
\end{aligned}$$

   (a) (10 marks) Derive the weak form of the above boundary value problem.

(b) (10 marks) Write down the shape functions of the element 3 that represent the nodes 6, 2, 7 respectively.

(c) (10 marks) Calculate the local element stiffness matrix (k^3_{ij}), $(i, j = 1, 2, 3)$.

(d) (10 marks) Calculate the surface integrals on the boundary $x = 0$ and $x = 8$.

(e) (10 marks) Use Poisson.m to find the finite element approximate solution of the boundary value problem. Plot the approximation and the exact solution $\Phi = xy$.