AM466/562: Finite Element Method

Quiz 2

April 5, 2005

Name: **ID Number:**

Undergraduate ——— **Graduate** ———

Score:

Problem 1 ————- Problem 2 ————

Total Score: —————

Note:

1. **Part 3 of Problem 2 is optional for undergraduates. You will NOT get any credit if you do it. So please indicate whether you are a undergraduate.**
2. No books and notes are allowed.
3. Show necessary steps.

Problem 1. Consider the Poisson's equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0, \quad (x,y) \in \Omega = (0,1) \times (0,1),$$

subject to the boundary conditions

$$\begin{aligned} u(1,y) &= 1, \quad \frac{\partial u}{\partial x}(0,y) = 0, \quad 0 \le y \le 1, \\ \frac{\partial u}{\partial y}(x,0) &= 0, \quad \frac{\partial u}{\partial y}(x,1) = 1 - u(x,1), \quad 0 \le x \le 1. \end{aligned}$$

1. (5 points) Derive the weak form.

2. (5 points) For the mesh of two triangular elements of the unit square, write three shape functions on the first element.

3. (5 points) The global element matrix K^2 on the second element from the domain part has been computed for you as follows

$$K^2 = \frac{1}{2}\begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 2 & -1 \\ 0 & 0 & -1 & 1 \end{pmatrix}.$$

For the global element matrix K^1 on the first element, four entries have been computed for you as follows

$$\begin{aligned} K^1_{11} &= \frac{1}{2}, \\ K^1_{12} &= -\frac{1}{2}, \end{aligned}$$

$$\begin{aligned} K^1_{14} &= 0, \\ K^1_{22} &= 1. \end{aligned}$$

Compute the rest of the entries and write the matrix K^1.

4. (5 points) For the global element matrix Σ^1_K on the first element from the boundary, one entry has been computed for you as follows

$$\Sigma^1_{K,22} = \frac{1}{3}.$$

Compute the rest of the entries and write the matrix Σ^1_K.

5. (5 points) Compute the element force vector on the first element from the boundary.

6. (5 points) Assemble the above element matrices and write the linear system for the nodal values u_1, u_2, u_3, u_4.

7. (5 points) Solve the system for nodal values.

8. (5 points) Write the FE approximate solution. Is the approximate solution equal to its exact solution?

Problem 2. For the six-node triangular master element shown in the following figure, five quadratic shape functions have been constructed for you as follows

$$\begin{aligned}
N_2(u,v) &= -4u(u+v-1),\\
N_3(u,v) &= u(2u-1),\\
N_4(u,v) &= 4uv,\\
N_5(u,v) &= v(2v-1),\\
N_6(u,v) &= -4(u+v-1)v.
\end{aligned}$$

1. (5 points) Construct the shape function $N_1(u,v)$ that represent the node 1.

2. (5 points) For a general element Ω_e shown in the above figure, which has the same nodes as the master element except node 4 (x_4, y_4), derive the isoparametric mapping, $x = x(u, v), y = y(u, v)$, which maps the master element into Ω_e.

3. (5 points, **Optional for undergraduates. No credit if you do it.**) Suppose that the quadratic polynomial

$$p_2(u, v) = a_1 + a_2 u + a_3 v + a_4 u^2 + a_5 uv + a_6 v^2,$$

vanishes on the side through the nodes 3 and 5. Show that p_2 can be written as the form

$$p_2(u, v) = N_1^L(u, v) p_1(u, v),$$

where $N_1^L(u, v) = 1 - u - v$ is the linear shape function on the three-node master element that represents node 1 and $p_1(u, v)$ is a linear function of u and v.