Review of Two Dimensional Problems

- **What have I taught**? See the notes.
- **You should be able to**
  1. derive the weak form of 2D PDEs;
  2. use the weak form to obtain FE approximation;
  3. derive the linear shape functions on triangular elements;
  4. derive the shape functions on master elements;
  5. construct parametric mappings on master elements and generate a parametric mesh;
  6. calculate element matrices;
  7. assemble element matrices;
  8. apply boundary conditions;
  9. use Gaussian quadrature to calculate numerical integrals;
  10. use software to generate a mesh;
  11. use and modify the programs poisson.m and steady.m and other FEM programs to solve two dimensional problems numerically;
  12. specify the boundary conditions and coefficients in the programs poisson.m and steady.m;
  13. analyze your numerical results;
  14. prove that the Galerkin's approximations is the best one.

- **What possible extensions should you continue to pursue?**

  1. Concentrated source problem: $H(x, y) = \delta(x - x_0, y - y_0)$ at a point (x_0, y_0)?
  2. Problems of two or more different materials. In this case the coefficients R_x and R_y are discontinuous across the interface of two different materials and then solutions are not twice differentiable across the interface.
  3. Error estimates in the case of high order elements.

- **Self-test problem** (Do the problem first before looking at the solutions). Consider the Poisson's equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0, \quad (x, y) \in \Omega = (0, 1) \times (0, 1),$$

  subject to the boundary conditions

$$\begin{aligned} u(1, y) &= 1, \quad \frac{\partial u}{\partial x}(0, y) = 0, \quad 0 \le y \le 1, \\ \frac{\partial u}{\partial x}(x, 0) &= 0, \quad \frac{\partial u}{\partial x}(x, 1) = 1 - u(x, 1), \quad 0 \le x \le 1. \end{aligned}$$

  1. Derive the weak form.
  2. For the mesh of one rectangular element of the unit square (the mesh will be drawn in class), write out the connectivity matrix for the mesh.

3. Write out four shape functions.
4. Compute the contribution to the stiffness matrix from the domain.
5. Compute the contribution to the stiffness matrix from the boundary.
6. Compute the force vector from the boundary.
7. Assemble these matrices and write out the linear system for the unknown nodal values.
8. solve the system to obtain nodal values.