Solutions of Self-test 1 of Chapter 3

1. **Chapter 3 Review Exercises 2.** $f(x) = \sqrt{x^2+3}$. $f'(x) = \frac{2x}{2\sqrt{x^2+3}}$, $f'(1) = \frac{2}{2\sqrt{1^2+3}} = 1/2$. So $\sqrt{x^2+3} \approx f(1) + f'(1)(x-1) = 2 + \frac{1}{2}(x-1)$.

2. **Chapter 3 Review Exercises 10.** $\lim_{x\to 0} \dfrac{\sin x}{x^2+3x} = \lim_{x\to 0} \dfrac{\frac{d}{dx}(\sin x)}{\frac{d}{dx}(x^2+3x)} = \lim_{x\to 0} \dfrac{\cos x}{2x+3} = 1/3$.

3. **Chapter 3 Review Exercises 12.** $\lim_{x\to\infty} \dfrac{x^2}{e^{3x}} = \lim_{x\to\infty} \dfrac{\frac{d}{dx}(x^2)}{\frac{d}{dx}(e^{3x})} = \lim_{x\to\infty} \dfrac{2x}{3e^{3x}} = \lim_{x\to\infty} \dfrac{\frac{d}{dx}(2x)}{\frac{d}{dx}(3e^{3x})} = \lim_{x\to\infty} \dfrac{2}{9e^{3x}} = 0$.

4. **Chapter 3 Review Exercises 24.** $f(x) = (x^2-1)^{2/3}$.

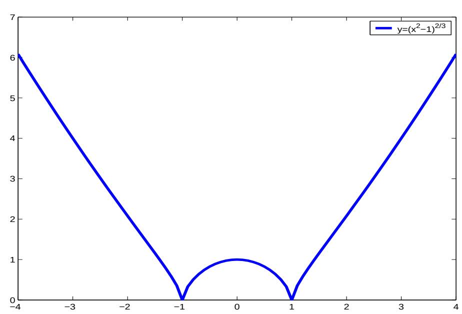
   (a) $f'(x) = \frac{4x}{3(x^2-1)^{1/3}}$. $f'(x) = 0$ at $x = 0$ and is undefined at $x = \pm 1$. So the critical numbers are $x = -1, 0$ and $x = 1$.

   (b) $f'(x)$ is negative for $x < -1$, positive for $-1 < x < 0$, negative for $0 < x < 1$, and positive for $x > 1$. So $f(x)$ is decreasing for $x < -1$, increasing for $-1 < x < 0$, decreasing for $0 < x < 1$, and increasing for $x > 1$.

   (c) Local minima at $x = \pm 1$ and local maximum at $x = 0$.

   (d) $f''(x) = \frac{4(x^2-3)}{9(x^2-1)^{4/3}}$. $f''(x) = 0$ at $x = \pm\sqrt{3}$ and is undefined at $x = \pm 1$. $f''(x)$ is positive for $x < -\sqrt{3}$, negative for $-\sqrt{3} < x < \sqrt{3}$, and positive for $x > \sqrt{3}$. So $f(x)$ is concave up for $x < -\sqrt{3}$, concave down for $-\sqrt{3} < x < \sqrt{3}$, and concave up for $x > \sqrt{3}$.

   (e) The inflection points are $x = \pm\sqrt{3}$.



5. **Chapter 3 Review Exercises 30.** $f(x) = x^2e^{-x}$ on [-1, 4].

   (a) Find all critical numbers: $f'(x) = 2xe^{-x} - x^2e^{-x} = xe^{-x}(2-x)$. Setting $f'(x) = 0$ gives the critical numbers $x = 0$ and 2.

   (b) Compute the values of the function at endpoints and critical numbers:

$$f(-1) = e, \quad f(0) = 0, \quad f(2) = 4/e^2, \quad f(4) = 16/e^4.$$

(c) Compare: $f(-1) = e$ is the absolute maximum and $f(0) = 0$ is the absolute minimum.

6. **Exercises 3.7: 15**. $V(x) = (10-2x)(6-2x)x$. $0 \le x \le 3$. $V'(x) = -2(6-2x)x - 2(10-2x)x + (10-2x)(6-2x) = 4(3x^2 - 16x + 15) = 0$. $V''(x) = 4(6x-16)$. $x = \frac{8}{3} \pm \frac{\sqrt{19}}{3}$. $x = \frac{8}{3} + \frac{\sqrt{19}}{3} > 3$ is beyond the range. $V''\left(\frac{8}{3} - \frac{\sqrt{19}}{3}\right) = 4\left(6\left(\frac{8}{3} - \frac{\sqrt{19}}{3}\right) - 16\right) < 0$. So V has the maximum at $x = \frac{8}{3} - \frac{\sqrt{19}}{3}$.

7. **Exercises 3.8: 9**. $x = 6, \frac{dx}{dt} = 3, y = \sqrt{10^2 - 6^2} = 8$.

$$10^2 = [x(t)]^2 + [y(t)]^2.$$

$$0 = 2x(t)\frac{dx}{dt} + 2y(t)\frac{dy}{dt}$$

$$\frac{dy}{dt} = -\frac{x}{y}\frac{dx}{dt} = -\frac{6}{8}(3) = -2.25 ft/s.$$