Self-test 2 of Chapter 3

Please work on the following problems and then grade them yourself. The solutions are on the other side. Please don't look at them before you do them yourself.

Exercises 3.2: 6, 16
Exercises 3.3: 32
Exercises 3.4: 12
Exercises 3.5: 2, 10
Exercises 3.7: 5
Exercises 3.8: 8

1. **Exercises 3.2: 6.** $\lim_{x\to 0} \frac{\sin x}{e^{3x}-1} = \lim_{x\to 0} \frac{\frac{d}{dx}(\sin x)}{\frac{d}{dx}(e^{3x}-1)} = \lim_{x\to 0} \frac{\cos x}{3e^{3x}} = 1/3.$

2. **Exercises 3.2: 16.** $\lim_{x\to\infty} \frac{e^{3x}}{x^4} = \lim_{x\to\infty} \frac{\frac{d}{dx}(e^{3x})}{\frac{d}{dx}(x^4)} = \lim_{x\to\infty} \frac{3e^{3x}}{4x^3} = \lim_{x\to\infty} \frac{\frac{d}{dx}(3e^{3x})}{\frac{d}{dx}(4x^3)} =$
$\lim_{x\to\infty} \frac{9e^{3x}}{12x^2} = \lim_{x\to\infty} \frac{\frac{d}{dx}(9e^{3x})}{\frac{d}{dx}(12x^2)} = \lim_{x\to\infty} \frac{27e^{3x}}{24x} = \lim_{x\to\infty} \frac{\frac{d}{dx}(27e^{3x})}{\frac{d}{dx}(24x)} = \lim_{x\to\infty} \frac{81e^{3x}}{24} = 81/24.$

3. **Exercises 3.3: 32.** $f(x) = x^4 - 8x^2 + 2$ on [-3, 1].
   (a) Find all critical numbers: $f'(x) = 4x^3 - 16x$. Setting $f'(x) = 0$ gives the critical numbers $x = 0, -2$ and 2.
   (b) Compute the values of the function at endpoints and critical numbers:
   $$f(-3) = 11, \quad f(-2) = -14, \quad f(0) = 2, \quad f(1) = -5.$$
   (c) Compare: $f(-3) = 11$ is the absolute maximum and $f(-2) = -14$ is the absolute minimum.

4. **Exercises 3.4, problem 12** $f(x) = x^5 - 5x^2 + 1$. $f'(x) = 5x^4 - 10x$. Critical numbers are $x = 0, \sqrt[3]{3}$. Since $f'(x)$ is positive on the interval $(-\infty, 0)$, negative on the interval $(0, \sqrt[3]{3})$, and positive on the interval $(\sqrt[3]{3}, \infty)$, $f(x)$ has a local maximum at $x = 0$ and a local minimum at $x = \sqrt[3]{3}$.

5. **Exercises 3.5, problem 2** $f(x) = x^4 - 6x^2 + 2x + 3$. $f'(x) = 4x^3 - 12x + 2$. $f''(x) = 12x^2 - 12$. $f''(x)$ is positive for $x < -1$, negative for $-1 < x < 1$, and positive for $x > 1$. So $f(x)$ is concave up for $x < -1$, concave down for $-1 < x < 1$, and concave up for $x > 1$.

6. **Exercises 3.5, problem 10** $f(x) = x^4 + 4x^2 + 1$. $f'(x) = 4x^3 + 8x$. Critical number is $x = 0$. $f''(x) = 12x^2 + 8$. $f''(0) = 8 > 0$. So it follows from the second derivative test $f(x)$ has a local minimum at $x = 0$

7. **Exercises 3.7: 5**. $A = xy$. $2x + 3y = 120$. $y = 40 - \frac{2x}{3}$. $A(x) = x\left(40 - \frac{2x}{3}\right)$. $A'(x) = 40 - \frac{4x}{3} = 0$. $x = 30$. $A''(x) = -\frac{4}{3}$. So the second derivative test insures that A has a maximum at $x = 30$. $y = 40 - \frac{2\cdot 30}{3} = 20$. Thus the dimensions are 20×30 feet.

8. **Exercises 3.8: 8**. $r = 200$, $\frac{dr}{dt} = 5$.
   $$A(t) = \pi[r(t)]^2.$$
   $$\frac{dA}{dt} = 2\pi r(t)\frac{dr}{dt}$$
   $$\frac{dA}{dt} = 2\pi \cdot 200 \cdot 5 = 2000\pi \; ft^2/min.$$