

Self-test 2 of Chapter 4

Please work on the following problems and then grade them yourself. The solutions are on the other side. Please don't look at them before you do them yourself.

Exercises 4.1: 10, 12, 18, 20, 22

Exercises 4.5: 30, 40

Exercises 4.6: 16, 18, 20, 22

Extra problem: Evaluate

$$\int \frac{x^2}{x^2 - 4} dx.$$

1. **Exercises 4.1: 10.** $\int \frac{x+2x^{3/4}}{x^{5/4}} dx = \int (x^{-1/4} + 2x^{-1/2}) dx = \frac{4}{3}x^{3/4} + 4x^{1/2} + c.$
2. **Exercises 4.1: 12.** $\int (3 \cos x - \sin x) dx = 3 \sin x + \cos x + c.$
3. **Exercises 4.1: 18.** $\int (4x - 2e^x) dx = 2x^2 - 2e^x + c.$
4. **Exercises 4.1: 20.** $\int (2x^{-1} + \sin x) dx = 2 \ln |x| - \cos x + c.$
5. **Exercises 4.1: 22.** $\int \frac{3}{4x^2+4} dx = \frac{3}{4} \tan^{-1} x + c.$
6. **Exercises 4.5: 30.** $f'(x) = \sin(x^2 + 1) \frac{d}{dx}(x^2 + 1) = 2x \sin(x^2 + 1).$
7. **Exercises 4.5: 40.** Set $x^2 - 4x = 0$. Then $x = 0, 4$. So $A = \int_0^4 (x^2 - 4x) dx = \left(\frac{1}{3}x^3 - 2x^2\right)_0^4 = \frac{1}{3}4^3 - 2 \cdot 4^2.$
8. **Exercises 4.6: 16.** $u = 1/x, du = -\frac{1}{x^2} dx$. So $\int \frac{\cos(1/x)}{x^2} dx = -\int \cos u du = -\sin u + c = -\sin(1/x) + c.$
9. **Exercises 4.6: 18.** $u = x^2 + 4, du = 2x dx$. So $\int \frac{x}{x^2+4} dx = \int \frac{1}{2} \frac{1}{u} du = \frac{1}{2} \ln |u| + c = \frac{1}{2} \ln |x^2 + 4| + c.$
10. **Exercises 4.6: 34.** $u = x^3, du = 3x^2 dx, u(0) = 0, u(2) = 8$. So $\int_0^2 x^2 e^{x^3} dx = \int_0^8 \frac{1}{3} e^u du = \frac{1}{3} e^u \Big|_0^8 = \frac{1}{3} (e^8 - 1).$
11. **Exercises 4.6: 40.** $u = x^2 + 1, du = 2x dx, u(0) = 1, u(1) = 2$. So $\int_0^1 \frac{x}{\sqrt{x^2+1}} dx = \int_1^2 \frac{1}{2} \frac{1}{\sqrt{u}} du = \sqrt{u} \Big|_1^2 = \sqrt{2} - 1.$