1. Exercises 2.7. 9. \( f(x) = 4^{-3x+1} \). \( f'(x) = 4^{-3x+1} \ln 4 \frac{d}{dx} (-3x+1) = -3 \cdot 4^{-3x+1} \ln 4 \).

2. Exercises 2.7. 19. \( f(x) = \sin[\ln(\cos x^3)] \). \( f'(x) = \cos[\ln(\cos x^3)] \frac{d}{dx} [\ln(\cos x^3)] = \cos[\ln(\cos x^3)] \frac{1}{\cos x} \) \( \cos(x^3) = \cos[\ln(\cos x^3)] \frac{1}{\cos x} (-\sin x^3) \frac{d}{dx} (x^3) = \cos[\ln(\cos x^3)] \frac{1}{\cos x} (-3x^2 \cos[\ln(\cos x^3)] \tan x^3) \).

3. Exercises 2.7. 39. \( f(x) = x^\sin x \). Apply the log: \( \ln f(x) = \ln x^\sin x = \sin x \ln x \). Then

\[
\frac{d}{dx} (\ln f(x)) = \frac{d}{dx} (\sin x \ln x),
\]

\[
\frac{f'(x)}{f(x)} = \frac{d}{dx} (\sin x \ln x) = \sin x \frac{d}{dx} (\ln x) + \frac{\sin x}{x},
\]

\[
f'(x) = x^\sin x \left( \cos x \ln x + \frac{\sin x}{x} \right).
\]

4. Exercises 2.8. 15. \( e^{4y} - \ln y = 2x \). Find \( y' \).

\[
\frac{d}{dx} (e^{4y} - \ln y) = \frac{d}{dx} (2x),
\]

\[
4e^{4y} y' - \frac{y'}{y} = 2,
\]

\[
4ye^{4y} y' - y' = 2y,
\]

\[
y' = \frac{2y}{4ye^{4y} - 1}.
\]

5. Exercises 2.8. 17.

\[
\frac{d}{dx} (x^2 - 4y^3) = \frac{d}{dx} (0),
\]

\[
2x - 12y^2 y' = 0,
\]

\[
y' = \frac{2x}{12y^2}.
\]

At (2,1):

\[
y'(2) = \frac{2 \cdot 2}{12 \cdot 1^2} = \frac{1}{3}.
\]

So the equation of the tangent line is:

\[
y = \frac{1}{3} (x - 2) + 1.
\]

6. Exercises 2.8. 29. \( f(x) = \tan^{-1} \sqrt{x} \). \( f'(x) = \frac{1}{1+(\sqrt{x})^2} \frac{d}{dx} (\sqrt{x}) = \frac{1}{1+x} \frac{1}{2} x^{-1/2} = \frac{1}{2 (1+x) \sqrt{x}} \).
7. **Exercises 2.9.5.** Since $f(0) = \sin 0 = 0 \neq f(\pi/2) = \sin(\pi/2) = 1$, the hypothesis of Rolle’s Theorem is not satisfied. But the hypothesis of the Mean Value Theorem is satisfied since $\sin x$ is continuous and differentiable on $[0, \pi/2]$. Since $f'(x) = \cos x$, the Mean Value Theorem gives

$$
\cos c = \frac{\sin(\pi/2) - \sin 0}{\pi/2 - 0} = \frac{2}{\pi},
$$

$$
c = \cos^{-1} \frac{2}{\pi}.
$$

8. **Exercises 2.9.7.** Proof. For any $a < b$, it follows from the Mean Value Theorem that there exist $c \in (a, b)$ such that

$$
\frac{f(b) - f(a)}{b - a} = f'(c) > 0,
$$

$$
f(b) - f(a) > 0,
$$

$$
f(a) < f(a).
$$

So $f(x)$ is increasing.

9. **Exercises 2.9.9.** $f(x) = x^3 + 5x + 1$. Since $f'(x) = 3x^2 + 5 > 0$, it follows from **Exercises 2.9.7** that $f(x)$ is increasing.