Solutions of Part of Assignment 6

1. **Exercises 2.7. 9.** $f(x) = 4^{-3x+1}$. $f'(x) = 4^{-3x+1}\ln 4\frac{d}{dx}(-3x+1) = -3\cdot 4^{-3x+1}\ln 4$.

2. **Exercises 2.7. 19.** $f(x) = \sin[\ln(\cos x^3)]$. $f'(x) = \cos[\ln(\cos x^3)]\frac{d}{dx}[\ln(\cos x^3)] = \cos[\ln(\cos x^3)]\frac{1}{\cos x^3}\frac{d}{dx}(\cos x^3) = \cos[\ln(\cos x^3)]\ \frac{1}{\cos x^3}(-\sin x^3)\frac{d}{dx}(x^3) = \cos[\ln(\cos x^3)]\ \frac{1}{\cos x^3}(-\sin x^3)3x^2 = -3x^2\cos[\ln(\cos x^3)]\tan x^3)$.

3. **Exercises 2.7. 39.** $f(x) = x^{\sin x}$. Apply the log: $\ln f(x) = \ln x^{\sin x} = \sin x \ln x$. Then
$$\frac{d}{dx}(\ln f(x)) = \frac{d}{dx}(\sin x \ln x),$$
$$\frac{f'(x)}{f(x)} = \frac{d}{dx}(\sin x)\ln x + \sin x\frac{d}{dx}(\ln x),$$
$$\frac{f'(x)}{f(x)} = \cos x \ln x + \frac{\sin x}{x},$$
$$f'(x) = x^{\sin x}\left(\cos x\ln x + \frac{\sin x}{x}\right).$$

4. **Exercises 2.8. 15.** $e^{4y} - \ln y = 2x$. Find y'.
$$\frac{d}{dx}(e^{4y} - \ln y) = \frac{d}{dx}(2x),$$
$$4e^{4y}y' - \frac{y'}{y} = 2,$$
$$4ye^{4y}y' - y' = 2y,$$
$$y' = \frac{2y}{4ye^{4y} - 1}.$$

5. **Exercises 2.8. 17.**
$$\frac{d}{dx}(x^2 - 4y^3) = \frac{d}{dx}(0),$$
$$2x - 12y^2y' = 0,$$
$$y' = \frac{2x}{12y^2}.$$
At (2,1):
$$y'(2) = \frac{2\cdot 2}{12\cdot 1^2} = \frac{1}{3}.$$
So the equation of the tangent line is:
$$y = \frac{1}{3}(x-2) + 1.$$

6. **Exercises 2.8. 29.** $f(x) = \tan^{-1}\sqrt{x}$. $f'(x) = \frac{1}{1+(\sqrt{x})^2}\frac{d}{dx}(\sqrt{x}) = \frac{1}{1+x}\frac{1}{2}x^{-1/2} = \frac{1}{2}\frac{1}{(1+x)\sqrt{x}}$.

7. **Exercises 2.9.5.** Since $f(0) = \sin 0 = 0 \neq f(\pi/2) = \sin(\pi/2) = 1$, the hypothesis of Rolle's Theorem is not satisfied. But the hypothesis of the Mean Value Theorem is satisfied since $\sin x$ is continuous and differentiable on $[0, \pi/2]$. Since $f'(x) = \cos x$, the Mean Value Theorem gives

$$\cos c = \frac{\sin(\pi/2) - \sin 0}{\pi/2 - 0} = \frac{2}{\pi},$$

$$c = \cos^{-1} \frac{2}{\pi}.$$

8. **Exercises 2.9.7.** Proof. For any $a < b$, it follows from the Mean Value Theorem that there exist $c \in (a, b)$ such that

$$\frac{f(b) - f(a)}{b - a} = f'(c) > 0,$$

$$f(b) - f(a) > 0,$$

$$f(a) < f(a).$$

So $f(x)$ is increasing.

9. **Exercises 2.9.9.** $f(x) = x^3 + 5x + 1$. Since $f'(x) = 3x^2 + 5 > 0$, it follows from **Exercises 2.9.7** that $f(x)$ is increasing.