Solutions of Part of Assignment 7

1. **Exercises 3.1. 5.** $f(x) = \sin 3x$. $f'(x) = 3\cos 3x, f'(0) = 3\cos 0 = 3$. So $\sin 3x \approx f'(0)(x-0) = 3x$.

2. **Exercises 3.1. 9.** $f(x) = \sqrt[4]{x}$. $f'(x) = \frac{1}{4}x^{-3/4}$. $f'(16) = \frac{1}{4}16^{-3/4} = 1/32$. So $\sqrt[4]{16.04} \approx f'(16)(16.04 - 16) = 0.4/32 = 1/80$.

3. **Exercises 3.2. 5.** $\displaystyle\lim_{x\to 0} \frac{e^{2x}-1}{x} = \lim_{x\to 0} \frac{\frac{d}{dx}(e^{2x}-1)}{\frac{d}{dx}(x)} = \lim_{x\to 0} \frac{2e^{2x}}{1} = 2.$

4. **Exercises 3.2. 15.** $\displaystyle\lim_{x\to\infty} \frac{x^3}{e^x} = \lim_{x\to\infty} \frac{\frac{d}{dx}(x^3)}{\frac{d}{dx}(e^x)} = \lim_{x\to\infty} \frac{3x^2}{e^x} = \lim_{x\to\infty} \frac{\frac{d}{dx}(3x^2)}{\frac{d}{dx}(e^x)} = \lim_{x\to\infty} \frac{6x}{e^x} =$ $\displaystyle\lim_{x\to\infty} \frac{\frac{d}{dx}(6x)}{\frac{d}{dx}(e^x)} = \lim_{x\to\infty} \frac{6}{e^x} = 0.$

5. **Exercises 3.2. 37.** Let $L = \displaystyle\lim_{x\to 0^+} (1/x)^x$. Then $\ln L = \ln \displaystyle\lim_{x\to 0^+} (1/x)^x = \lim_{x\to 0^+} \ln(1/x)^x =$ $\displaystyle\lim_{x\to 0^+} x\ln(1/x) = \lim_{x\to 0^+} x(-\ln x) = \lim_{x\to 0^+} \frac{-\ln x}{\frac{1}{x}} = \lim_{x\to 0^+} \frac{-\frac{d}{dx}(\ln x)}{\frac{d}{dx}\left(\frac{1}{x}\right)} = \lim_{x\to 0^+} \frac{-\frac{1}{x}}{-\frac{1}{x^2}} =$ $\displaystyle\lim_{x\to 0^+} x = 0$. So $\displaystyle\lim_{x\to 0^+} (1/x)^x = L = 1$.

6. **Exercises 3.3.7.** $f(x) = x^3 - 3x + 1$. Set $f'(x) = 3x^2 - 3 = 0$. Solving it gives the critical numbers $x = -1$ and $x = 1$. Local minimum at 1 and local maximum at -1.

7. **Exercises 3.3.17.** $f(x) = \frac{x^2-2}{x+2}$. $f'(x) = \frac{\frac{d}{dx}(x^2-2)(x+2)-(x^2-2)\frac{d}{dx}(x+2)}{(x+2)^2} = \frac{2x(x+2)-(x^2-2)}{(x+2)^2} = \frac{x^2+4x+2}{(x+2)^2}$. Setting $f'(x) = 0$ gives $x^2 + 4x + 2 = 0$. Solving it gives the critical numbers $x = -2 + \sqrt{2}$ and $x = -2 - \sqrt{2}$. Local minimum at $x = -2 + \sqrt{2}$ and local maximum at $x = -2 - \sqrt{2}$.

8. **Exercises 3.3.33.** $f(x) = x^{2/3}$ on [-4, -2].

   (a) Find all critical numbers: $f'(x) = \frac{2}{3}x^{-1/3}$. Since $f'(x)$ is undefined at $x = 0$ and $f(x)$ is defined at $x = 0$, 0 is the critical number.

   (b) Compute the values of the function at endpoints and critical numbers:

   $$f(-4) = (-4)^{2/3} = 4^{2/3}, \quad f(-2) = (-2)^{2/3} = 2^{2/3}, \quad f(0) = 0^{2/3} = 0.$$

   (c) Compare: $f(-4) = 4^{2/3}$ is the absolute maximum and $f(0) = 0$ is the absolute minimum.