Solutions of part of Assignment 8

1. **Exercises 3.4, problem 1** $f(x) = x^3 - 3x + 2$. $f'(x) = 3x^2 - 3$. Critical numbers are $x = \pm 1$. Since $f'(x) > 0$ on the intervals $(-\infty, -1)$ and $(1, \infty)$, $f(x)$ is increasing. Since $f'(x) < 0$ on the interval $(-1, 1)$, $f(x)$ is decreasing.

2. **Exercises 3.4, problem 11** $f(x) = x^4 + 4x^3 - 2$. $f'(x) = 4x^3 + 12x^2$. Critical numbers are $x = 0, -3$. Since $f'(x)$ is positive on the interval $(-3, \infty)$ and negative on the interval $(-\infty, -3)$, $f(x)$ has a local minimum at $x = -3$ and no extremum at $x = 0$.

3. **Exercises 3.5, problem 7** $f(x) = x^{4/3} + 4x^{1/3}$. $f'(x) = \frac{4}{3}x^{1/3} + \frac{4}{3}x^{-2/3}$. $f''(x) = \frac{4}{9}x^{-2/3} - \frac{8}{9}x^{-5/3} = \frac{4}{9x^{2/3}}\left(1 - \frac{2}{x}\right)$. Since the quantity $\frac{4}{9x^{2/3}}$ is never negative, the sign of the second derivative is the same as the sign of $1 - \frac{2}{x}$. So $f''(x)$ is positive for $x > 2$, negative for $x < 2$. So $f(x)$ is concave up for $x > 2$, concave down for $x < 2$.

4. **Exercises 3.5, problem 9** $f(x) = x^4 + 4x^3 - 1$. $f'(x) = 4x^3 + 12x^2$. Critical numbers are $x = 0, -3$. $f''(x) = 12x^2 + 24x$. $f''(0) = 0$. So the second derivative test is inconclusive. Since $f'(x)$ does not change sign when x moving across 0, by the first derivative test, $f(x)$ has no extremum at 0. Since $f''(-3) = 36$, it follows from the second derivative test $f(x)$ has a local minimum at $x = -3$

5. **Exercises 3.7: 5**. $A = xy$. $2x + 3y = 120$. $y = 40 - \frac{2x}{3}$. $A(x) = x\left(40 - \frac{2x}{3}\right)$. $A'(x) = 40 - \frac{4x}{3} = 0$. $x = 30$. $A''(x) = -\frac{4}{3}$. So the second derivative test insures that A has a maximum at $x = 30$. $y = 40 - \frac{2\cdot 30}{3} = 20$. Thus the dimensions are 20×30 feet.

6. **Exercises 3.8: 5**. $r = 3, \frac{dr}{dt} = 1$.

$$A(t) = \pi[r(t)]^2.$$

$$\frac{dA}{dt} = 2\pi r(t)\frac{dr}{dt}$$

$$\frac{dA}{dt} = 2\pi \cdot 3 \cdot 1 = 6\pi\ mm^2/hr.$$