Solutions of part of Assignment 8

1. **Exercises 3.4, problem 1** \( f(x) = x^3 - 3x + 2 \). \( f'(x) = 3x^2 - 3 \). Critical numbers are \( x = \pm 1 \). Since \( f'(x) > 0 \) on the intervals \((-\infty, -1)\) and \((1, \infty)\), \( f(x) \) is increasing. Since \( f'(x) < 0 \) on the interval \((-1, 1)\), \( f(x) \) is decreasing.

2. **Exercises 3.4, problem 11** \( f(x) = x^4 + 4x^3 - 2 \). \( f'(x) = 4x^3 + 12x^2 \). Critical numbers are \( x = 0, -3 \). Since \( f'(x) \) is positive on the interval \((-3, \infty)\) and negative on the interval \((-\infty, -3)\), \( f(x) \) has a local minimum at \( x = -3 \) and no extremum at \( x = 0 \).

3. **Exercises 3.5, problem 7** \( f(x) = x^{4/3} + 4x^{1/3} \). \( f'(x) = \frac{4}{3}x^{1/3} + \frac{4}{3}x^{-2/3} \). \( f''(x) = \frac{4}{9}x^{-2/3} - \frac{8}{9}x^{-5/3} = \frac{4}{9x^{5/3}} \left(1 - \frac{2}{3}\right) \). Since the quantity \( \frac{4}{9x^{5/3}} \) is never negative, the sign of the second derivative is the same as the sign of \( 1 - \frac{2}{3} \). So \( f''(x) \) is positive for \( x > 2 \), negative for \( x < 2 \). So \( f(x) \) is concave up for \( x > 2 \), concave down for \( x < 2 \).

4. **Exercises 3.5, problem 9** \( f(x) = x^4 + 4x^3 - 1 \). \( f'(x) = 4x^3 + 12x^2 \). Critical numbers are \( x = 0, -3 \). \( f''(x) = 12x^2 + 24x \). \( f''(0) = 0 \). So the second derivative test is inconclusive. Since \( f'(x) \) does not change sign when \( x \) moving across 0, by the first derivative test, \( f(x) \) has no extremum at 0. Since \( f''(-3) = 36 \), it follows from the second derivative test \( f(x) \) has a local minimum at \( x = -3 \).

5. **Exercises 3.7**: 5. \( A = xy \). \( 2x + 3y = 120 \). \( y = 40 - \frac{2x}{3} \). \( A(x) = x \left(40 - \frac{2x}{3}\right) \). \( A'(x) = 40 - \frac{4x}{3} = 0 \). \( x = 30 \). \( A''(x) = -\frac{4}{3} \). So the second derivative test insures that \( A \) has a maximum at \( x = 30 \). \( y = 40 - \frac{2 \cdot 30}{3} = 20 \). Thus the dimensions are \( 20 \times 30 \) feet.

6. **Exercises 3.8**: 5. \( r = 3, \frac{dr}{dt} = 1 \).

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A(t) = \pi[r(t)]^2.
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\frac{dA}{dt} = 2\pi r(t) \frac{dr}{dt}
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\[
\frac{dA}{dt} = 2\pi \cdot 3 \cdot 1 = 6\pi \text{ mm}^2/\text{hr}.
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