Solutions of part of Assignment 9

1. **Exercises 4.1, problem 9** $\int \frac{x^{1/3}-3}{x^{2/3}}dx = \int (x^{-1/3} - 3x^{-2/3})dx = \int x^{-1/3}dx - 3\int x^{-2/3}dx = \frac{1}{1-1/3}x^{2/3} + \frac{3}{1-2/3}x^{1/3} + c = \frac{3}{2}x^{2/3} + 9x^{1/3} + c$.

2. **Exercises 4.1, problem 19** $\int (3\cos x - 1/x)dx = 3\int \cos x dx - \int 1/x dx = 3\sin x - \ln|x| + c$.

3. **Exercises 4.1, problem 27** $\int \frac{e^x+3}{e^x}dx = \int dx + 3\int e^{-x}dx = x - 3e^{-x} + c.$

4. **Exercises 4.2, problem 13** $\sum_{i=1}^{100}(i^2 - 3i + 2) = \sum_{i=1}^{100} i^2 - 3\sum_{i=1}^{100} i + \sum_{i=1}^{100} 2$

   $= \frac{100(100+1)(200+1)}{6} - 3 \cdot \frac{100(100+1)}{2} + 200.$

5. **Exercises 4.2, problem 19** $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{1}{n}\left[\left(\frac{i}{n}\right)^2 + 2\left(\frac{i}{n}\right)\right]$

   $= \lim_{n\to\infty}\left[\sum_{i=1}^{n}\frac{1}{n}\left(\frac{i}{n}\right)^2 + \sum_{i=1}^{n}\frac{2}{n}\left(\frac{i}{n}\right)\right] = \lim_{n\to\infty}\left[\frac{1}{n^3}\sum_{i=1}^{n} i^2 + \frac{2}{n^2}\sum_{i=1}^{n} i\right]$

   $= \lim_{n\to\infty}\left[\frac{1}{n^3}\frac{n(n+1)(2n+1)}{6} + \frac{2}{n^2}\frac{n(n+1)}{2}\right] = \frac{1}{6}\left(1 + \lim_{n\to\infty}\frac{1}{n}\right)\left(2 + \lim_{n\to\infty}\frac{1}{n}\right) + 1 +$
   $\lim_{n\to\infty}\frac{1}{n} = \frac{1}{3} + 1.$

6. **Exercises 4.3, problem 15** $A = \lim_{n\to\infty} \sum_{i=1}^{n} \frac{1}{n}\left[\left(\frac{i}{n}\right)^2 + 1\right]$

   $= \lim_{n\to\infty}\left[\sum_{i=1}^{n}\frac{1}{n}\left(\frac{i}{n}\right)^2 + \sum_{i=1}^{n}\frac{1}{n}\right] = \lim_{n\to\infty}\left[\frac{1}{n^3}\sum_{i=1}^{n} i^2 + \frac{1}{n}\sum_{i=1}^{n} 1\right]$

   $= \lim_{n\to\infty}\left[\frac{1}{n^3}\frac{n(n+1)(2n+1)}{6} + \frac{1}{n}n\right] = \frac{1}{6}\left(1 + \lim_{n\to\infty}\frac{1}{n}\right)\left(2 + \lim_{n\to\infty}\frac{1}{n}\right) + 1 = \frac{1}{3} + 1.$

7. **Exercises 4.4, problem 7** $\int_0^2 x^2 dx = \lim_{n\to\infty}\sum_{i=1}^{n}\frac{2}{n}\left(\frac{2i}{n}\right)^2 = \lim_{n\to\infty}\frac{8}{n^3}\sum_{i=1}^{n} i^2$

   $= \lim_{n\to\infty}\frac{8}{n^3}\frac{n(n+1)(2n+1)}{6} = \frac{8}{6}\left(1 + \lim_{n\to\infty}\frac{1}{n}\right)\left(2 + \lim_{n\to\infty}\frac{1}{n}\right) = \frac{8}{3}.$

8. **Exercises 4.4, problem 47** $\int_0^4 f(x)dx = \int_0^1 2x dx + \int_1^4 4dx = 1 + 12 = 13.$