MATH 1592 - Final Review

1 Chapter 7

1.1 Main Topics

1. Integration techniques:
   - Fitting integrands to basic rules on page 485.
   - Integration by parts, Theorem 7.1 on page 488.
   - Guidelines for trigonometric integrals on pages 497 and 500.
   - Trigonometric substitutions on page 506.
   - Partial fractions on page 516.
2. L'Hôpital's Rules for computation of limits on page 531.
3. Definitions of improper integrals on pages 540 and 543.

1.2 Review Exercises

Do the exercises on pages 550 and 551: 1, 5, 7, 10, 12, 13, 17, 18, 20, 22, 23, 25, 27, 31, 33, 71, 73, 75, 79, 81.

2 Chapter 8

2.1 Main Topics

1. Definition of the limit of a sequence on page 557.
2. Definition of the convergence of a series on page 567.
3. Techniques of limit computations:
   - Non-existence of oscillating sequence: $\{(-1)^n + 2\}$.
   - change a sequence to a function, example 4 on page 558.
   - Squeeze Theorem on page 559.
   - Absolute Value Theorem on page 560.
   - Bounded and monotonic sequences on page 563.
4. Convergence of special Series:

- Geometric Series on page 569.
- p-series on page 579.
- Telescoping series on page 568.

5. Tests for series convergence:

   - nth-term test on page 571.
   - The integral test on page 577.
   - Limit comparison test on page 585.
   - Alternating series test on page 590.
   - The ration test on page 597.
   - The root test on page 600.

6. Strategies for testing series on page 601.

7. Important sequence limits used frequently:

   - $\lim_{n\to\infty} \sqrt[n]{n} = 1$.
   - $\lim_{n\to\infty} \dfrac{\ln n}{n} = 0$.
   - $\lim_{n\to\infty} x^n = 0 \quad (|x| < 10)$.
   - $\lim_{n\to\infty} x^{1/n} = 1 \quad (x > 0)$.
   - $\lim_{n\to\infty} \left(1 + \dfrac{x}{n}\right)^n = e^x$ (any x).
   - $\lim_{n\to\infty} \dfrac{x^n}{n!} = 0$ (any x).

8. Taylor polynomial on page 607.

9. Methods of finding Radius and interval of convergence of a power series, examples on page 618 and 619.

   - By Geometric power series.
   - By the root test.
   - By the ratio test.
   - check endpoints for the interval of convergence.

10. Methods of finding a power series expansion of a function:

    - By Geometric power series, example 1 on page 626.
    - By integration, example 5 on page 629
    - By Differentiation, example 8 on page 622.
    - By direct substitution, example 6 on page 638.
    - By partial fractions, example 3 on page 627.
    - By Taylor series, example 1 on page 633.

2.2 Review Exercises

Page 643: 5, 9, 13, 15, 23, 29, 33, 37, 41, 45, 57, 65, 73, 79, 83, 97.

3 Chapter 9

3.1 Main Topics

1. Definitions and standard equations of parabola, ellipse, and hyperbola (Section 9.1).
2. Definition of parametric equations on page 665.
3. Conversion between parametric equations and rectangular equations
4. Parametric form of derivatives on page 675.
5. Arc length in Parametric form (page 678) and polar form (page 698).
6. Area of a surface of revolution in Parametric form (page 680) and polar form (page 699).
7. Polar coordinates on page 684.
8. Coordinate conversion on page 685.
9. Sketch of polar graphs on page 686.
10. Slope in polar form on page 688.
11. Area of a polar region.
12. Parameter equations used frequently:
    - Circle:
      $$x = h + a\cos\theta, \quad y = k + a\sin\theta, \quad 0 \le \theta < 2\pi.$$
    - Ellipse:
      $$x = h + a\cos\theta, \quad y = k + b\sin\theta, \quad 0 \le \theta < 2\pi.$$
    - Hyperbola:
      $$x = h + a\sec\theta, \quad y = k + a\tan\theta.$$
    - Cycloid:
      $$x = a(\theta - \sin\theta), \quad y = a(1 - \cos\theta).$$
13. Polar graphs used frequently:
    - Circle:
      $$r = a, \quad \text{or } r = a\sin\theta, \quad \text{or } r = a\cos\theta.$$
    - Line:
      $$\theta = \alpha, \quad \text{or } r = \sec\theta.$$
    - Rose:
      $$r = a\sin(n\theta), \quad \text{or } r = a\cos(n\theta).$$
    - Limaçons:
      $$r = a \pm b\sin\theta, \quad \text{or } r = a \pm b\cos\theta \quad a > 0, b > 0.$$

3.2 Review Exercises

Review all problems handed out on Fridays and the Review Exercises for Chapter 9 on page 709: 5, 6, 7, 8, 23, 24, 25, 26, 27, 30, 31, 37, 43, 49, 51, 53, 55, 60, 61, 65, 67, 69, 73, 81, 90, 95.

4 Chapter 10

4.1 Main Topics

1. Vectors:
   - Component form.
   - Standard unit vectors $\mathbf{i}, \mathbf{j}, \mathbf{k}$.
   - norm.
   - Algebraic operation.
   - Dot product: $\mathbf{u} \cdot \mathbf{v} = u_1v_1 + u_2v_2 + u_3v_3$.
   - Cross product: $\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix}$
   - Angle between two vectors: $\cos\theta = \frac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{u}\|\|\mathbf{v}\|}$.
   - Parallel vectors: $\mathbf{u} = c\mathbf{v}$.
   - Orthogonal: $\mathbf{u} \cdot \mathbf{v} = 0$.
   - $\|\mathbf{u} \times \mathbf{v}\| = \|\mathbf{u}\|\|\mathbf{v}\| \sin\theta$, the area of parallelogram having $\mathbf{u}$ and $\mathbf{v}$ as adjacent sides.
   - $\mathbf{u} \times \mathbf{v}$ is orthogonal to both $\mathbf{u}$ and $\mathbf{v}$ sides.
   - Direction Cosines.
   - Applications: force, velocity, torque, work, and so on.
2. Space Coordinates and distance formula.
3. Surfaces in space:
   - The line equation: $x = x_1 + at$, $y = y_1 + bt$, $z = z_1 + ct$ (parametric) and $\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$ (symmetric) where the direction vector $\mathbf{v} = \langle a, b, c\rangle$ is parallel to the line.
   - The plane equation: $a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$ or $ax + by + cz + d = 0$, where the normal vector $\mathbf{n} = \langle a, b, c\rangle$ is orthogonal to the plane.
   - The angle between two planes: $\cos\theta = \frac{|\mathbf{n_1}\cdot\mathbf{n_2}|}{\|\mathbf{n_1}\|\|\mathbf{n_2}\|}$, where $\mathbf{n_1}$ and $\mathbf{n_2}$ are two normal vectors of the planes.
   - Distance between a point Q and a plane: $D = \frac{|\overrightarrow{PQ}\cdot\mathbf{n}|}{\|\mathbf{n}\|}$, where P is a point in the plane and $\mathbf{n}$ is normal to the plane.
   - Distance between a point Q and a line: $D = \frac{|\overrightarrow{PQ}\times\mathbf{u}|}{\|\mathbf{u}\|}$, where P is a point one the line and $\mathbf{u}$ is a direction vector for the line.

- Sphere: $(x-x_0)^2+(y-y_0)^2+(z-z_0)^2=r^2$.
- Cylinder.
- General quadric surface: $AX^2+By^2+Cz^2+Dxy+Exz+Fyz+Gx+Hy+Iz+J=0$.
- Surfaces of revolution.

4. Conversion between cylindrical and rectangular coordinates:

$$x=r\cos\theta,\ y=r\sin\theta,\ z=z.$$

5. Conversion between spherical and rectangular coordinates:

$$x=\rho\sin\phi\cos\theta,\ y=\rho\sin\phi\sin\theta,\ z=\rho\cos\phi.$$

4.2 Review Exercises

Review all homework problems and the Review Exercises for Chapter 10 on page 780: 1, 3, 5, 7, 12, 15, 17, 19, 21, 23, 25, 27, 30, 35, 39, 41, 43, 45, 47, 48, 49, 52, 53, 55, 59, 63, 65, 67, 69.

5 Chapter 11

5.1 Main Topics

1. Limits, differentiation, and integration of vector-valued functions.
2. Velocity $\mathbf{v}(t)=\mathbf{r}'(t)$.
3. Acceleration $\mathbf{a}(t)=\mathbf{r}''(t)$.
4. Tangent vector $\mathbf{T}(t)=\frac{\mathbf{r}'(t)}{\|\mathbf{r}'(t)\|}$.
5. Normal vector $\mathbf{N}(t)=\frac{\mathbf{T}'(t)}{\|\mathbf{T}'(t)\|}$.
6. Arc length $s=\int_a^b\|\mathbf{r}'(t)\|dt$.
7. Curvature $K=\|\mathbf{T}'(s)\|$.

5.2 Review Exercises

Page 832: 3, 7, 21, 23, 27, 29, 35, 37, 41, 47, 53, 58, 63, 67.