MATH 1592 - Review of Chapter 10

1 Main Topics

1. Vectors:
   - Component form.
   - Standard unit vectors $\mathbf{i}, \mathbf{j}, \mathbf{k}$.
   - norm.
   - Algebraic operation.
   - Dot product: $\mathbf{u} \cdot \mathbf{v} = u_1v_1 + u_2v_2 + u_3v_3$.
   - Cross product: $\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix}$
   - Angle between two vectors: $\cos\theta = \frac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{u}\|\|\mathbf{v}\|}$.
   - Parallel vectors: $\mathbf{u} = c\mathbf{v}$.
   - Orthogonal: $\mathbf{u} \cdot \mathbf{v} = 0$.
   - $\|\mathbf{u} \times \mathbf{v}\| = \|\mathbf{u}\|\|\mathbf{v}\| \sin\theta$, the area of parallelogram having $\mathbf{u}$ and $\mathbf{v}$ as adjacent sides.
   - $\mathbf{u} \times \mathbf{v}$ is orthogonal to both $\mathbf{u}$ and $\mathbf{v}$ sides.
   - Direction Cosines.
   - Applications: force, velocity, torque, work, and so on.
2. Space Coordinates and distance formula.
3. Surfaces in space:
   - The line equation: $x = x_1 + at$, $y = y_1 + bt$, $z = z_1 + ct$ (parametric) and $\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$ (symmetric) where the direction vector $\mathbf{v} = \langle a, b, c\rangle$ is parallel to the line.
   - The plane equation: $a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$ or $ax + by + cz + d = 0$, where the normal vector $\mathbf{n} = \langle a, b, c\rangle$ is orthogonal to the plane.
   - The angle between two planes: $\cos\theta = \frac{|\mathbf{n_1}\cdot\mathbf{n_2}|}{\|\mathbf{n_1}\|\|\mathbf{n_2}\|}$, where $\mathbf{n_1}$ and $\mathbf{n_2}$ are two normal vectors of the planes.
   - Distance between a point Q and a plane: $D = \frac{|\overrightarrow{PQ}\cdot\mathbf{n}|}{\|\mathbf{n}\|}$, where P is a point in the plane and $\mathbf{n}$ is normal to the plane.
   - Distance between a point Q and a line: $D = \frac{|\overrightarrow{PQ}\times\mathbf{u}|}{\|\mathbf{u}\|}$, where P is a point one the line and $\mathbf{u}$ is a direction vector for the line.
   - Sphere: $(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2 = r^2$.
   - Cylinder.
   - General quadric surface: $AX^2+By^2+Cz^2+Dxy+Exz+Fyz+Gx+Hy+Iz+J = 0$.
   - Surfaces of revolution.

4. Conversion between cylindrical and rectangular coordinates:

$$x = r\cos\theta,\ y = r\sin\theta,\ z = z.$$

5. Conversion between spherical and rectangular coordinates:

$$x = \rho\sin\phi\cos\theta,\ y = \rho\sin\phi\sin\theta,\ z = \rho\cos\phi.$$

2 Review Exercises

Review all homework problems and the Review Exercises for Chapter 10 on page 780: 1, 3, 5, 7, 12, 15, 17, 19, 21, 23, 25, 27, 30, 35, 39, 41, 43, 45, 47, 48, 49, 52, 53, 55, 59, 63, 65, 67, 69.