MATH 1591 - Solution of Review of Chapter 7

Evaluate the following integrals:

1. $\int x\sqrt{x^2-1}\,dx = \frac{1}{2}\int \sqrt{x^2-1}\,d(x^2-1) = \frac{1}{2}\cdot\frac{2}{3}(x^2-1)^{3/2} = \frac{1}{3}(x^2-1)^{3/2}$

2. $\int x^2\sin 2x\,dx = -\frac{1}{2}x^2\cos 2x + \int x\cos 2x\,dx = -\frac{1}{2}x^2\cos 2x + \frac{1}{2}x\sin 2x - \frac{1}{2}\int \sin 2x\,dx = -\frac{1}{2}x^2\cos 2x + \frac{1}{2}x\sin 2x + \frac{1}{4}\cos 2x + C.$

   Integration by parts twice: (1) $dv = \sin 2xdx, v = -\frac{1}{2}\cos 2x; u = x^2, du = 2xdx$. (2) $dv = \cos 2xdx, v = \frac{1}{2}\sin 2x; u = x, du = dx$.

3. $\int \frac{-12}{x^2\sqrt{4-x^2}}\,dx$. $x = 2\sin\theta, dx = 2\cos\theta d\theta$. $\int \frac{-12}{x^2\sqrt{4-x^2}}\,dx = \int \frac{-24\cos\theta d\theta}{4\sin^2\theta(2\cos\theta)} = -3\int \csc^2\theta d\theta = 3\cot\theta + C = \frac{3\sqrt{4-x^2}}{x} + C$

4. $\int_0^{16} \frac{1}{\sqrt[4]{x}}\,dx = \lim_{b\to 0^+} \frac{4}{3}x^{3/4}\Big|_b^{16} = \frac{32}{3}$

Use L'Hôpital's Rules to evaluate the limits:

1. $\lim_{x\to 0} \frac{\sin \pi x}{\sin 2\pi x} = \lim_{x\to 0} \frac{\pi\cos \pi x}{2\pi\cos 2\pi x} = \frac{\pi\cos \pi 0}{2\pi\cos 2\pi 0} = \frac{1}{2}$

2. $\lim_{x\to\infty} \frac{e^{2x}}{x^2} = \lim_{x\to\infty} \frac{2e^{2x}}{2x} = \lim_{x\to\infty} \frac{2e^{2x}}{1} = \infty$