Solutions of Problem 4

Determine the convergence of the following series. If convergent and possible, find their sums:

1. $\sum_{n=0}^{\infty} 2(-1.03)^n$.

   This is a geometric series. It diverges since $-1.03 < -1$.

2. $\sum_{n=0}^{\infty} 5\left(-\frac{1}{3}\right)^n$.

   This is a geometric series. It converges and its sum is

$$\sum_{n=0}^{\infty} 5\left(-\frac{1}{3}\right)^n = \frac{5}{1+\frac{1}{3}} = \frac{15}{4}.$$

3. $\sum_{n=1}^{\infty} \frac{1}{(n+1)(n+2)}$

   This is a telescoping series.

$$\frac{1}{(n+1)(n+2)} = \frac{1}{n+1} - \frac{1}{n+2}$$

$$\sum_{n=1}^{\infty} \frac{1}{(n+1)(n+2)} = \frac{1}{2} - \frac{1}{3} + \frac{1}{3} - \frac{1}{4} + \frac{1}{4} - \frac{1}{5} + \cdots = \frac{1}{2}.$$

4. $\sum_{n=1}^{\infty} \frac{n}{\sqrt{n^2+1}}$

   Since

$$\lim_{n\to\infty} \frac{n}{\sqrt{n^2+1}} = \lim_{n\to\infty} \frac{1}{\sqrt{1+\frac{1}{n^2}}} = \frac{1}{\sqrt{1+\lim_{n\to\infty}\frac{1}{n^2}}} = 1$$

   By the n-th term test, it diverges.

5. $\sum_{n=1}^{\infty} \frac{1}{n}$

   This is the harmonic series and it diverges.

6. $\sum_{n=1}^{\infty} \frac{1}{n^{1.00001}}$

   This is p-series. It converges since $p = 1.00001 > 1$.

7. $\frac{\ln 2}{2} + \frac{\ln 3}{3} + \frac{\ln 4}{4} + \frac{\ln 5}{5} + \frac{\ln 6}{6} + \cdot$ Since

$$\int_2^{\infty} \frac{\ln x}{x} dx = \int_2^{\infty} \ln x d\ln x = \frac{1}{2}(\ln x)^2|_2^{\infty} = \infty,$$

   by the integral test, the series diverges.