

Problem 7

Please work on the following problems:

1. Section 9.2: 24, 30, 42, 48.
2. Section 9.3: 6, 10, 42, 51

Solutions are on the other side.

Solutions

1. **Section 9.2: 24.** $x = 4 + 2 \cos \theta, y = -1 + 2 \sin \theta; \implies (x-4)^2 = 4 \cos^2 \theta, (y+1)^2 = 4 \sin^2 \theta; \implies (x-4)^2 + (y+1)^2 = 4.$
2. **Section 9.2: 30.** $x = \ln(2t), y = t^2; \implies t = \frac{e^x}{2}; \implies y = \frac{e^{2x}}{4}.$
3. **Section 9.2: 42.** $x = h + a \sec \theta, y = k + b \tan \theta; \implies \frac{x-h}{a} = \sec \theta, \frac{y-k}{b} = \tan \theta; \implies \frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1.$
4. **Section 9.2: 48.** $a = (7 - (-3))/2 = 5, c = (5 - (-1))/2 = 3, b = \sqrt{a^2 - c^2} = 4;$
Center: $((4, 5) + (4, -1))/2 = (4, 2).$ So, $x = 4 + 5 \cos \theta, y = 2 + 4 \sin \theta.$
5. **Section 9.3: 6.** $x = \sqrt{t}, y = 3t - 1.$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{3}{1/(2\sqrt{t})} = 6\sqrt{t}.$$

$$\frac{d^2y}{dx^2} = \frac{3/\sqrt{t}}{1/(2\sqrt{t})} = 6.$$

So when $t = 1,$

$$\frac{dy}{dx} = 6, \quad \frac{d^2y}{dx^2} = 6.$$

6. **Section 9.3: 10.** $x = \cos \theta, y = 3 \sin \theta.$

$$\frac{dy}{dx} = \frac{3 \cos \theta}{-\sin \theta}.$$

$$\frac{d^2y}{dx^2} = \frac{3 \csc^2 \theta}{-\sin \theta} = -\frac{3}{\sin^3 \theta}.$$

So when $\theta = 0,$ $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2} = 6$ are undefined.

7. **Section 9.3: 42.** $x = a \cos \theta, y = a\theta.$

$$s = 4 \int_0^{\pi/2} \sqrt{a^2 \sin^2 \theta + a^2 \cos^2 \theta} d\theta = 4a \int_0^{\pi/2} d\theta = 2\pi a.$$

8. **Section 9.3: 51.** $x = 4 \cos \theta, y = 4 \cos \theta. x' = -4 \sin \theta, y' = 4 \cos \theta.$

$$S = 2\pi \int_0^{\pi/2} 4 \cos \theta \sqrt{4^2 \sin^2 \theta + 4^2 \cos^2 \theta} d\theta = 32\pi \int_0^{\pi/2} \cos \theta d\theta = 32\pi.$$