

Solutions of Problem Sheet 1

(1/20/06)

Evaluate the following indefinite integrals by fitting integrands to basic rules:

1. $\int x \left(1 + \frac{1}{x}\right)^3 dx = \int x \left(1 + \frac{3}{x} + \frac{3}{x^2} + \frac{1}{x^3}\right) dx = \int \left(x + 3 + \frac{3}{x} + \frac{1}{x^2}\right) dx = \frac{1}{2}x^2 + 3x + 3 \ln|x| - \frac{1}{x} + C.$
2. $\int \frac{2x}{x-4} dx = \int \frac{2x-8+8}{x-4} dx = \int 2 dx + \int \frac{8}{x-4} dx = 2x + 8 \ln|x-4| + C.$
3. $\int \frac{1}{(x-1)\sqrt{4x^2-8x+3}} dx = \int \frac{2}{2(x-1)\sqrt{[2(x-1)]^2-1}} dx = \operatorname{arcsec}|2(x-1)| + C.$
4. $\int x\sqrt{4-2x^2} dx. u = 4-2x^2, du = -4xdx. \int x\sqrt{4-2x^2} dx = -\frac{1}{4} \int (4-2x^2)^{1/2} d(4-2x^2) dx = -\frac{1}{6}(4-2x^2)^{3/2} + C.$
5. $\int_1^e \frac{1-\ln x}{x} dx = \int_1^e \frac{1}{x} dx - \int_1^e \frac{\ln x}{x} dx = \ln x \Big|_1^e - \int_1^e \ln x d \ln x = 1 - \frac{1}{2}(\ln x)^2 \Big|_1^e = \frac{1}{2}.$
6. $\int \frac{5}{3e^x-2} dx = \int \frac{5}{3e^x-2} \frac{e^{-x}}{e^{-x}} dx = 5 \int \frac{e^{-x}}{3-2e^{-x}} dx = \frac{5}{2} \int \frac{1}{3-2e^{-x}} d(3-2e^{-x}) = \frac{5}{2} \ln|3-2e^{-x}| + C.$