Evaluate the following integrals

1. \[ \int x^2 \ln x \, dx = \frac{1}{3} \int \ln x \, d(x^3) = \frac{1}{3} x^3 \ln x - \frac{1}{3} \int x^3 d(\ln x) = \frac{1}{3} x^3 \ln x - \frac{1}{3} \int x^2 dx = \frac{1}{3} x^3 \ln x - \frac{1}{9} x^3 + C. \]

2. \[ \int x^2 e^{3x} \, dx = \frac{1}{3} \int x^2 \, d(e^{3x}) = \frac{1}{3} x^2 e^{3x} - \frac{1}{3} \int x^2 e^{3x} \, dx = \frac{1}{3} x^2 e^{3x} - \frac{2}{9} x e^{3x} + \frac{2}{27} e^{3x} + C. \]

3. \[ \int \cos^3 x \sin^4 x \, dx = \int \cos^2 x \sin^4 x \cos x \, dx = \int (1 - \sin^2 x) \sin^4 x d\sin x = \int (\sin^4 x - \sin^6 x) \sin x = \frac{1}{5} \sin^5 x - \frac{1}{7} \sin^7 x + C. \]

4. \[ \int \frac{x^2}{\sqrt{25-x^2}} \, dx. \quad x = 5 \sin \theta. \quad dx = 5 \cos \theta d\theta. \quad \int \frac{x^2}{\sqrt{25-x^2}} \, dx = \int \frac{25 \sin^2 \theta}{\sqrt{25-25 \sin^2 \theta}} 5 \cos \theta d\theta = 25 \int \frac{\sin^2 \theta \cos \theta}{\sqrt{\cos^2 \theta}} d\theta = 25 \int \sin^2 \theta \, d\theta = 25 \int \frac{1 - \cos(2\theta)}{2} \, d\theta = \frac{25}{2} (\theta - \frac{1}{2} \sin(2\theta)) + C = \frac{25}{2} \left( \arcsin \left( \frac{x}{5} \right) - \frac{x}{5} \right) \left( \sqrt{25-x^2} \right) + C = \frac{1}{2} \left( 25 \arcsin \left( \frac{x}{5} \right) - x \sqrt{25-x^2} \right) + C. \]