Problem Sheet 2

Evaluate the following integrals

1. $\int x^2 \ln x \ \ dx = \frac{1}{3}\int \ln x \ \ d(x^3) = \frac{1}{3}x^3 \ln x - \frac{1}{3}\int x^3 d(\ln x) = \frac{1}{3}x^3 \ln x - \frac{1}{3}\int x^2 dx = \frac{1}{3}x^3 \ln x - \frac{1}{9}x^3 + C.$

2. $\int x^2 e^{3x} \ dx = \frac{1}{3}\int x^2 \ d(e^{3x}) = \frac{1}{3}x^2 e^{3x} - \frac{1}{3}\int e^{3x} d(x^2) = \frac{1}{3}x^2 e^{3x} - \frac{1}{3}\int 2x e^{3x} dx = \frac{1}{3}x^2 e^{3x} - \frac{1}{9}\int 2x d(e^{3x}) = \frac{1}{3}x^2 e^{3x} - \frac{2}{9}x e^{3x} + \frac{2}{9}\int e^{3x} dx = \frac{1}{3}x^2 e^{3x} - \frac{2}{9}x e^{3x} + \frac{2}{27}e^{3x} + C.$

3. $\int \cos^3 x \sin^4 x \ dx = \int \cos^2 x \sin^4 x \cos x \ dx = \int (1 - \sin^2 x) \sin^4 x d \sin x = \int (\sin^4 x - \sin^6 x) d \sin x = \frac{1}{5}\sin^5 x - \frac{1}{7}\sin^7 x + C.$

4. $\int \frac{x^2}{\sqrt{25-x^2}} \ dx$. $x = 5\sin\theta$. $dx = 5\cos\theta d\theta$. $\int \frac{x^2}{\sqrt{25-x^2}} \ dx = \int \frac{25\sin^2\theta}{\sqrt{25-25\sin^2\theta}} \ 5\cos\theta d\theta = 25\int \frac{\sin^2\theta\cos\theta}{\sqrt{\cos^2\theta}} \ d\theta = 25\int \sin^2\theta \ d\theta = 25\int \frac{1-\cos(2\theta)}{2} \ d\theta = \frac{25}{2}(\theta - \frac{1}{2}\sin(2\theta)) + C = \frac{25}{2}(\theta - \sin\theta\cos\theta) + C = \frac{25}{2}\left(\arcsin\left(\frac{x}{5}\right) - \left(\frac{x}{5}\right)\left(\frac{\sqrt{25-x^2}}{5}\right)\right) + C = \frac{1}{2}\left(25\arcsin\left(\frac{x}{5}\right) - x\sqrt{25-x^2}\right) + C$