

Math 1592 Solutions of Quiz 2

(20 points, 5 each) Evaluate the following integrals:

1. $\int \sin^5(2x) \cos(2x) dx$. $u = \sin 2x, du = 2 \cos 2x dx$. $\int \sin^5(2x) \cos(2x) dx = \frac{1}{2} \int u^5 du = \frac{1}{12} u^6 = \frac{1}{12} \sin^6 2x$.
2. $\int_0^{\pi/2} \cos^3 x dx$. $u = \sin x, du = \cos x dx$. $\int_0^{\pi/2} \cos^2 x \cos x dx = \int_0^{\pi/2} (1 - \sin^2 x) \cos x dx = (\sin x - \frac{1}{3} \sin^3 x)_0^{\pi/2} = \frac{2}{3}$.
3. $\int \frac{1}{(25-x^2)^{3/2}} dx$ $x = 5 \sin \theta, dx = 5 \cos \theta d\theta$. $\int \frac{1}{(25-x^2)^{3/2}} dx = \int \frac{1}{(25-25 \sin^2 \theta)^{3/2}} 5 \cos \theta d\theta = \int \frac{5 \cos \theta}{(5 \cos \theta)^3} d\theta = \frac{1}{25} \int \sec^2 \theta d\theta = \frac{1}{25} \tan \theta + C = \frac{x}{25\sqrt{25-x^2}} + C$.
4. $\int \frac{1}{(1+x^2)^{3/2}} dx$ $x = \tan \theta, dx = \sec^2 \theta d\theta$. $\int \frac{1}{(1+\tan^2 \theta)^{3/2}} \sec^2 \theta d\theta = \int \frac{1}{\sec^3 \theta} \sec^2 \theta d\theta = \int \frac{1}{\sec \theta} d\theta = \int \cos \theta d\theta = \sin \theta + C = \frac{x}{\sqrt{x^2+1}} + C$.