Math 1592 Solutions of Quiz 3

Problem 1. (10 points, 5 each) Determine the convergence of the following sequences. If convergent, find the limits:

1. $a_n = \frac{5n^2}{n^2+2}$.

   Method 1: Use the L'Hôpital's Rule.

$$\lim_{n\to\infty} \frac{5n^2}{n^2+2} = \lim_{x\to\infty} \frac{5x^2}{x^2+2} = \lim_{x\to\infty} \frac{10x}{2x} = 5.$$

   Method 2: Use limit properties.

$$\lim_{n\to\infty} \frac{5n^2}{n^2+2} = \lim_{n\to\infty} \frac{5}{1+\frac{2}{n^2}} = \frac{5}{1+\lim\limits_{n\to\infty} \frac{2}{n^2}} = 5.$$

2. $a_n = (-1)^n$.

   Diverges because the sequence alternates between -1 and 1.

Problem 2. (10 points, 5 each) Determine the convergence of the following series. If convergent, find their sums:

1. $\sum\limits_{n=0}^{\infty} 2\left(\frac{3}{4}\right)^n$.

   This is a geometric series. Because $3/4 < 1$, it is convergent and its sum is

$$\sum_{n=0}^{\infty} 2\left(\frac{3}{4}\right)^n = \frac{2}{1-\frac{3}{4}} = 8$$

2. $\sum\limits_{n=1}^{\infty} \frac{n^2}{n^2+1}$

   Since

$$\lim_{n\to\infty} \frac{n^2}{n^2+1} = \lim_{n\to\infty} \frac{1}{1+\frac{1}{n^2}} = \frac{1}{1+\lim\limits_{n\to\infty} \frac{1}{n^2}} = 1 \neq 0,$$

   the series diverges.