Math 1592 Solutions of Quiz 4

(20 points, 5 each) Determine the convergence or divergence of the following series:

1. $\displaystyle\sum_{n=1}^{\infty}\frac{1}{n+1}$

   Integral Test. Since

   $$\int_1^{\infty}\frac{1}{x+1}dx = \ln(1+x)_1^{\infty} = \infty,$$

   it diverges.

2. $\displaystyle\sum_{n=1}^{\infty}\frac{n}{n^2+1}$

   Limit Comparison Test. Since

   $$\lim_{n\to\infty}\frac{\frac{n}{n^2+1}}{\frac{1}{n}} = \lim_{n\to\infty}\frac{n^2}{n^2+1} = 1,$$

   and $\displaystyle\sum_{n=1}^{\infty}\frac{1}{n}$ diverges, $\displaystyle\sum_{n=1}^{\infty}\frac{n}{n^2+1}$ diverges.

3. $\displaystyle\sum_{n=1}^{\infty}(-1)^n\frac{1}{n}$

   Alternating Series Test. This is a alternating series. Since

   $$\lim_{n\to\infty}\frac{1}{n} = 0$$

   and $\{\frac{1}{n}\}$ is decreasing, $\displaystyle\sum_{n=1}^{\infty}(-1)^n\frac{1}{n}$ converges.

4. $\displaystyle\sum_{n=1}^{\infty}\frac{2^n}{n^2}$

   Root Test. Since

   $$\lim_{n\to\infty}\sqrt[n]{\frac{2^n}{n^2}} = \lim_{n\to\infty}\frac{2}{n^{2/n}} = 2 > 1,$$

   $\displaystyle\sum_{n=1}^{\infty}\frac{2^n}{n^2}$ diverges.

   we can also use **Ratio Test.**