MATH 3110 - Review of Chapter 3

1 Main Topics

1.1 Important Assumptions and Physical Laws

1. The rate of population growth is proportional to the total population.

2. The relative rate of population growth is linearly dependent on the total population, that is,
$$\frac{dP/dt}{P} = aP + b.$$

3. One-compartment system: the rate of change of the amount x in the system is equal to input rate r_{in} subtracted by output rate r_{out}:
$$\frac{dx}{dt} = r_{in} - r_{out}.$$

4. Law of Mass Action: the rate of reaction is proportional to the product of concentrations of chemicals.

5. Newton's second law of motion:
$$F = ma.$$

1.2 Important Models

1. The Malthusian model:
$$\frac{dP}{dt} = kP, \quad P(0) = P_0.$$

2. Logistic model:
$$\frac{dP}{dt} = P(a - bP), \quad P(0) = P_0.$$

3. Lotka-Volterra predator-prey model:
$$\begin{aligned} \frac{dx}{dt} &= -ax + bxy, \\ \frac{dy}{dt} &= cy - dxy, \\ x(0) &= x_0,\ y(0) = y_0. \end{aligned}$$

4. The first-order reaction model:
$$\frac{dx}{dt} = kx, \quad x(0) = x_0.$$

5. The second-order reaction model:

$$\frac{dx}{dt} = x(a - bx), \quad x(0) = x_0.$$

6. The falling body model:

$$m\frac{d^2s}{dt^2} = mg - k\frac{ds}{dt}, \quad s(0) = s_0,\ s'(0) = s_1.$$

7. The spring-mass system:

$$m\frac{d^2x}{dt^2} + \beta\frac{dx}{dt} + \omega^2 x = 0, \quad x(0) = x_0,\ x'(0) = x_1.$$

8. **The mixture model:**

$$\frac{dx}{dt} = r_{in} - r_{out}, \quad x(0) = x_0.$$

2 Review Problems

1. Section 1.3: 9, 10, 11.
2. Section 3.1: 19, 24
3. Chapter 3 in Review (page 122): 16