MATH 3110 - Review of Chapter 4

1 Main Topics

1. Fundamental set of solutions (Section 4.1).
2. Linear independence/dependence. (Section 4.1).
3. The method of reduction of order (Section 4.2).
4. Methods of solving higher-order homogeneous linear equations with constant coefficients (Section 4.3):

$$ay'' + by' + cy = 0$$
$$am^2 + bm + c = 0$$

   - Distinct real roots m_1 and m_2:
   $$y = c_1 e^{m_1 x} + c_2 e^{m_2 x};$$
   - Repeated real roots $m_1 = m_2$:
   $$y = c_1 e^{m_1 x} + c_2 x e^{m_1 x};$$
   - Conjugate complex roots $m_1 = \alpha + i\beta$ and $m_2 = \alpha - i\beta$:
   $$y = e^{\alpha x}(c_1 \cos(\beta x) + c_2 \sin(\beta x));$$
   - Similar formulas for higher-order equations.

5. The method of undetermined coefficients (Section 4.4):

$$ay'' + by' + cy = f(x)$$

$$am^2 + bm + c = 0 \tag{1}$$

$$f(x) = (p_n x^n + \cdots + p_1 x + p_0)e^{\lambda x}\cos(\mu x) + (q_l x^l + \cdots + q_1 x + q_0)e^{\lambda x}\sin(\mu x)$$

   The form of a particular solution:

$$y_p(x) = x^r(A_k x^k + \cdots + A_1 x + A_0)e^{\lambda x}\cos(\mu x) + x^r(B_k x^k + \cdots + B_1 x + B_0)e^{\lambda x}\sin(\mu x)$$

   where

   - $k = \max(n, l)$,
   - $r = 0$ if $\lambda + i\mu$ is not a root of (1)
   - or $r = j$ if $\lambda + i\mu$ is a root of (1) with the multiplicity j.

6. The method of variation of parameters (Section 4.6):

$$y'' + P(x)y' + Q(x)y = f(x)$$

- Step 1. Find the general solutions of the associated homogeneous equations

$$y'' + P(x)y' + Q(x)y = 0. \tag{2}$$

- Step 2. Determine a particular solution of the form:

$$y_p(x) = u_1(x)y_1(x) + u_2(x)y_2(x)$$

where y_1 and y_2 are solutions of (2 and

$$u_1' = \frac{W_1}{W}, \quad u_2' = \frac{W_2}{W},$$

$$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}, \quad W_1 = \begin{vmatrix} 0 & y_2 \\ f(x) & y_2' \end{vmatrix}, \quad W_2 = \begin{vmatrix} y_1 & 0 \\ y_1' & f(x) \end{vmatrix}.$$

- Step 3. The general solutions are

$$y(x) = c_1y_1(x) + c_2y_2(x) + u_1(x)y_1(x) + u_2(x)y_2(x).$$

7. The substitution $x = e^t$ to solve Cauchy-Euler equations $ax^2y'' + bxy' + cy = f(x)$ (Section 4.7).

8. Boundary value problems:

- Step 1. Find general solutions.
- Step 2. Use boundary conditions to determine the constants in the general solutions.

9. Initial value problems:

- Step 1. Find general solutions.
- Step 2. Use initial conditions to determine the constants in the general solutions.

2 Review Problems

Chapter 4 in Review (page 189): 2, 3, 6 (a), (b), (e), (h), 8, 9, 10, 11, 13, 15, 16, 17, 18, 20, 21, 23, 25, 29, 31.