Math 1592 Solutions of Quiz 6

Problem 1. (5 points) Eliminate the parameter and write the corresponding rectangular equation for the following parametric equations:

\[ x = 4 \sin(2\theta), \quad y = 2 \cos(2\theta). \]

\[ \frac{x^2}{16} = \sin^2(2\theta), \quad \frac{y^2}{4} = \cos^2(2\theta). \]

\[ \frac{x^2}{16} + \frac{y^2}{4} = 1. \]

Problem 2. (5 points) Find \( \frac{dy}{dx} \) and \( \frac{d^2y}{dx^2} \) for

\[ x = t + 1, \quad y = t^2 + 3t. \]

\[ \frac{dy}{dx} = \frac{dy}{dt} \frac{dt}{dx} = \frac{2t + 3}{1} = 2t + 3. \]

\[ \frac{d^2y}{dx^2} = \frac{d}{dx} \left( \frac{dy}{dx} \right) = \frac{2}{1} = 2. \]

Problem 3. (5 points) Plot the point \((4, \pi/2)\) in polar coordinates and find its corresponding rectangular coordinates.

\[ x = 4 \cos(\pi/2) = 0, \quad y = 4 \sin(\pi/2) = 4. \]

So \((x, y) = (0, 4)\).

Problem 4. (5 points) Convert the polar equation to rectangular form and sketch its graph:

\[ r = \sin \theta. \]

\[ r = 5 \cos \theta \implies r^2 = r \sin \theta \implies x^2 + y^2 = y \implies x^2 + y^2 - y + \frac{1}{4} = \frac{1}{4} \implies \]

\[ x^2 + \left(y - \frac{1}{2}\right)^2 = \left(\frac{1}{2}\right)^2. \]