

Math 1592 Solutions of Quiz 6

Problem 1. (5 points) Eliminate the parameter and write the corresponding rectangular equation for the following parametric equations:

$$x = 4 \sin(2\theta), \quad y = 2 \cos(2\theta).$$

$$\frac{x^2}{16} = \sin^2(2\theta), \quad \frac{y^2}{4} = \cos^2(2\theta).$$

$$\frac{x^2}{16} + \frac{y^2}{4} = 1.$$

Problem 2. (5 points) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ for

$$x = t + 1, \quad y = t^2 + 3t.$$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2t + 3}{1} = 2t + 3.$$

$$\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}} = \frac{2}{1} = 2.$$

Problem 3. (5 points) Plot the point $(4, \pi/2)$ in polar coordinates and find its corresponding rectangular coordinates.

$$x = 4 \cos(\pi/2) = 0, \quad y = 4 \sin(\pi/2) = 4.$$

So $(x, y) = (0, 4)$.

Problem 4. (5 points) Convert the polar equation to rectangular form and sketch its graph:

$$r = \sin \theta.$$

$$r = 5 \cos \theta \implies r^2 = r \sin \theta \implies x^2 + y^2 = y \implies x^2 + y^2 - y + \frac{1}{4} = \frac{1}{4} \implies$$

$$x^2 + \left(y - \frac{1}{2}\right)^2 = \left(\frac{1}{2}\right)^2.$$