Math 1592 Solutions of Quiz 7

Problem 1. (5 points) Find the component form of the vector $\mathbf{v}$ and sketch the vector with its initial point at the origin.

Problem 2. (5 points) Given that $\mathbf{u} = \langle \mathbf{1}, \mathbf{2}, \mathbf{3} \rangle, \mathbf{v} = \langle \mathbf{2}, \mathbf{2}, -\mathbf{1} \rangle$, and $\mathbf{w} = \langle \mathbf{4}, \mathbf{0}, -\mathbf{4} \rangle$, find the vector $\mathbf{z} = \mathbf{2u} + \mathbf{4v} - \mathbf{w}$ and its length $\|\mathbf{z}\|$.

$$\mathbf{z} = \mathbf{2}\langle \mathbf{1}, \mathbf{2}, \mathbf{3} \rangle + \mathbf{4}\langle \mathbf{2}, \mathbf{2}, -\mathbf{1} \rangle - \langle \mathbf{4}, \mathbf{0}, -\mathbf{4} \rangle = \langle \mathbf{2}+\mathbf{8}-\mathbf{4}, \mathbf{4}+\mathbf{8}-\mathbf{0}, \mathbf{6}-\mathbf{4}-(-\mathbf{4}) \rangle = \langle \mathbf{6}, \mathbf{12}, \mathbf{6} \rangle.$$

$$\|\mathbf{z}\| = \sqrt{\mathbf{6^2} + \mathbf{12^2} + \mathbf{6^2}} = \sqrt{\mathbf{216}}.$$

Problem 3. (5 points) Given that $\mathbf{u} = \mathbf{2i} - \mathbf{j} + \mathbf{k}$ and $\mathbf{v} = \mathbf{i} - \mathbf{k}$, find $\mathbf{u} \cdot (\mathbf{2v})$ and $(\mathbf{u} \cdot \mathbf{v})\mathbf{v}$.

$$\mathbf{u} \cdot (\mathbf{2v}) = \langle \mathbf{2}, -\mathbf{1}, \mathbf{1} \rangle \cdot (\mathbf{2}\langle \mathbf{1}, \mathbf{0}, -\mathbf{1} \rangle) = \langle \mathbf{2}, -\mathbf{1}, \mathbf{1} \rangle \cdot \langle \mathbf{2}, \mathbf{0}, -\mathbf{2} \rangle = \mathbf{2} \cdot \mathbf{2} + (-\mathbf{1}) \cdot \mathbf{0} + \mathbf{1} \cdot (-\mathbf{2}) = \mathbf{2}.$$

$$(\mathbf{u} \cdot \mathbf{v})\mathbf{v} = (\langle \mathbf{2}, -\mathbf{1}, \mathbf{1} \rangle \cdot \langle \mathbf{1}, \mathbf{0}, -\mathbf{1} \rangle)\mathbf{v} = (\mathbf{2} \cdot \mathbf{1} + (-\mathbf{1}) \cdot \mathbf{0} + \mathbf{1} \cdot (-\mathbf{1}))(\mathbf{i} - \mathbf{k}) = \mathbf{i} - \mathbf{k}.$$

Problem 4. (5 points) Find the angle θ between two vectors $\mathbf{u} = \langle \mathbf{1}, \mathbf{1} \rangle$ and $\mathbf{v} = \langle \mathbf{2}, -\mathbf{2} \rangle$.

$$\cos\theta = \frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u}\|\|\mathbf{v}\|} = \frac{0}{\sqrt{2}\sqrt{8}} = 0.$$

$$\theta = \pi/2.$$