Project Topics of Math 3331 – Differential Equations

1 Riccati Equation

Riccati equation are as follows

$$y' = a(x)y + b(x)y^2 + c(x).$$

1. Discuss the existence and uniqueness of their solutions
2. Try to solve them in some special simple cases analytically
3. Solve them numerically.
4. Try to transform them to other equations like in the handouts.
5. Analyze the stability of thir equilibria.
6. Do any more you like.

2 Basic Enzyme Reaction

One of the most basic enzyme reactions can be represented by

$$S + E \underset{k_2}{\overset{k_1}{\rightleftharpoons}} C \xrightarrow{k_3} P + E,$$

where S denotes a substrate, E an enzyme, C a complex formed from S and E, P a product, and k_1, k_2, k_3 constants of reaction.

Law of Mass Action: The rate of a reaction is proportional to the product of the concentrations of the reactants.

Pseudo-Steady State Assumption: The reaction for the complex C is essentially in a steady state. Mathematically this implies that $\frac{dC}{dt} = 0$.

1. Use the above law to build a mathematical model for the reaction.
2. Try solve the model analytically and numerically.
3. Find the equilibria of the system.
4. Analyze the stability of these equilibria.
5. Do whatever you like.

3 Population Growth with Food Supply

The population growth with food supply can be modeled mathematically by the differential equations:

$$\frac{dP}{dt} = P\left(r - \frac{r}{K}P\right) + k_1 PS, \quad (1)$$

$$\frac{dS}{dt} = k_2 P - k_3 PS, \quad (2)$$

where P denotes the total population, S denotes the food supply, and r, K, k_1, k_2, k_3 are positive constants.

1. First consider the case of infinity carrying capacity K:

$$\frac{dP}{dt} = rP + k_1 PS, \quad (3)$$

$$\frac{dS}{dt} = k_2 P - k_3 PS, \quad (4)$$

2. Try to solve the model analytically
3. Find the equilibria of the system.
4. Analyze the stability of these equilibria.
5. Repeat the above for the original nonlinear one.
6. Do whatever you like.