10.1098/rspa.2001.0789

THE ROYAL SOCIETY

A reaction–diffusion model for a single species with age structure. I. Travelling-wave fronts on unbounded domains

By Joseph W.-H. So[1], Jianhong Wu[2] and Xingfu Zou[3]

[1]*Department of Mathematical Sciences, University of Alberta, Edmonton, Alberta, Canada T6G 2G1*
[2]*Department of Mathematics and Statistics, York University, Toronto, Ontario, Canada M3J 1P3*
[3]*Department of Mathematics and Statistics, Memorial University of Newfoundland, St John's, Newfoundland, Canada A1C 5S7*

Received 21 September 2000; accepted 29 January 2001

In this paper, we derive the equation for a single species population with two age classes and a fixed maturation period living in a spatially unbounded environment. We show that if the mature death and diffusion rates are age independent, then the total mature population is governed by a reaction–diffusion equation with time delay and non-local effect. We also consider the existence, uniqueness and positivity of solution to the initial-value problem for this type of equation. Moreover, we establish the existence of a travelling-wave front for the special case when the birth function is the one which appears in the well-known Nicholson's blowflies equation and we consider the dependence of the minimal wave speed on the mobility of the immature population.

Keywords: delay; diffusion; structured population; non-local effect; travelling waves

1. Introduction

When incorporating diffusion into a time delay model, many investigators generally simply add a diffusion term to the corresponding delay ordinary differential equation (ODE) model (Yoshida 1982; Memory 1989; Yang & So 1998; Feng & Lu 1999). But in recent years it has become recognized that there are modelling difficulties with this approach. The problem is that individuals have not been at the same point in space at previous times. It appears that the first comprehensive attempt to address this difficulty was made by Britton (1990), who addressed the problem for a delayed Fisher equation on an infinite spatial domain. His idea was that, to account for the drift of individuals to their present position from all possible positions at previous times, the delay term has to involve a weighted spatial averaging over the whole of the infinite domain, the weighting to be properly derived using probabilistic arguments and the assumptions being made about the motion of the individuals. Gourley & Britton (1996) developed the ideas further by studying a predator–prey system with time delays and associated spatial averaging, again on an infinite domain, and they developed a general approach to the study of linear stability of the uniform steady

© 2001 The Royal Society