Assignment 11

1. Solve the initial value problem

$$\frac{\partial^2 u}{\partial t^2} = 4\frac{\partial^2 u}{\partial x^2},$$
$$u(x,0) = e^{-x^2}, \quad \frac{\partial u}{\partial t}(x,0) = \cos x.$$

2. If

$$u(x,t) = F(x-ct) + G(x+ct).$$

then

$$u(x_0,t_0) - u(x_0+c\xi, t_0+\xi) - u(x_0-c\eta, t_0+\eta) + u(x_0+c\xi-c\eta, t_0+\xi+\eta) = 0$$

for any x_0 and t_0. Geometrically, for any parallelogram $A(x_0,t_0), B(x_0+c\xi, t_0+\xi), C(x_0+c\xi-c\eta, t_0+\xi+\eta), D(x_0-c\eta, t_0+\eta)$ we have

$$u(A) + u(C) = u(B) + u(D). \tag{1}$$

Solve the initial boundary value problem

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2},$$
$$u(0,t) = u(\pi,t) = 0,$$
$$u(x,0) = 1, \quad \frac{\partial u}{\partial t}(x,0) = 0$$

by (a) using (1), and (b) Fourier method (separation of variables). Discuss whether the solutions agree.