Assignment 11

1. Solve the initial value problem

\[
\frac{\partial^2 u}{\partial t^2} = 4 \frac{\partial^2 u}{\partial x^2},
\]

\[
u(x, 0) = e^{-x^2}, \quad \frac{\partial u}{\partial t}(x, 0) = \cos x.
\]

2. If

\[
u(x, t) = F(x - ct) + G(x + ct).
\]

then

\[
u(x_0, t_0) - u(x_0 + c\xi, t_0 + \xi) - u(x_0 - c\eta, t_0 + \eta) + u(x_0 + c\xi - c\eta, t_0 + \xi + \eta) = 0
\]

for any \(x_0\) and \(t_0\). Geometrically, for any parallelogram \(A(x_0, t_0), B(x_0 + c\xi, t_0 + \xi), C(x_0 + c\xi - c\eta, t_0 + \xi + \eta), D(x_0 - c\eta, t_0 + \eta)\) we have

\[
u(A) + \nu(C) = \nu(B) + \nu(D). \tag{1}
\]

Solve the initial boundary value problem

\[
\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2},
\]

\[
u(0, t) = \nu(\pi, t) = 0,
\]

\[
u(x, 0) = 1, \quad \frac{\partial u}{\partial t}(x, 0) = 0
\]

by (a) using (1), and (b) Fourier method (separation of variables). Discuss whether the solutions agree.