Published in *Comm. Partial Differential Equations* 23(1&2) (1998), 201-221.

LOCAL BOUNDARY CONTROLLABILITY FOR THE SEMILINEAR PLATE EQUATION

Weijiu Liu

Department of Mathematics, University of Wollongong
Wollongong, NSW 2522, Australia
E-mail: weijiu_liu@uow.edu.au

Abstract. The problem of local controllability for the semilinear plate equation with Dirichlet boundary conditions is studied. By making use of Schauder's fixed point theorem and the inverse function theorem, we prove that this system is locally controllable under a super-linear assumption on the nonlinearity, that is, the initial states in a small neighborhood of 0 in a certain function space can be driven to rest by Dirichlet boundary controls. Our super-linear assumption includes the critical exponent.

1. Introduction and Main Result

In this paper we are concerned with the problem of local controllability for the semilinear plate equation with Dirichlet boundary control:

$$\begin{cases} y'' + \Delta^2 y + \mu y + f(y) = 0 & \text{in } Q, \\ y(0) = y^0, \quad y'(0) = y^1 & \text{in } \Omega, \\ y = 0, \quad \dfrac{\partial y}{\partial \nu} = \phi \chi_{\Sigma_0} & \text{on } \Sigma. \end{cases} \tag{1.1}$$

In (1.1), Ω is a bounded domain (nonempty, open, and connected) in $\mathbb{R}^n$ with suitably smooth boundary $\Gamma = \partial\Omega$ (say C^3). $Q = \Omega \times (0,T)$ and $\Sigma = \Gamma \times (0,T)$ where $T > 0$. ν is the unit normal of Γ pointing towards the exterior of Ω. $y' = \dfrac{\partial y}{\partial t}$, $y(0) = y(x,0)$, $y'(0) = y'(x,0)$. $f(y)$ is a given function, χ_{Σ_0} denotes the characteristic function of a subset Σ_0 of Σ, and $\phi\chi_{\Sigma_0}$ represents a controller acting on the part of Σ given by Σ_0.

Let $\mathcal{C}$ be the set of all initial states (y^0, y^1) in a suitable Hilbert space (to be specified later), each of which can be steered to rest by a controller ϕ, that is, the solution of (1.1) also satisfies

$$y(x,T;\phi) = 0, \qquad y'(x,T;\phi) = 0 \qquad \text{in } \Omega. \tag{1.2}$$

The set $\mathcal{C}$ is called *the set of null controllability.*

Definition 1.1. *The system (1.1) is said to be locally controllable if the set $\mathcal{C}$ of null controllability contains an open neighborhood of 0 in a suitable Hilbert space.*

Definition 1.1 follows the definition of local controllability for a control process in $I\!R^n$ (see [8, p.364]).

Problems of local controllability have been studied for long time. The earliest studies (see [8, 12]) are concerned with the nonlinear control process defined by a differential system in $I\!R^n$

$$x' = f(t, x, u),$$

where $x(t) : [0, T] \to I\!R^n$ is the response for the control process, $u(t) : [0, T] \to I\!R^m$ is a controller, and f is a given function in $I\!R^{n+m+1}$. The method used to treat this process was the theory of nonlinear ordinary differential equations together with an inverse function type approach developed in [8]. Subsequently, this inverse function type approach was used to consider the local controllability for the nonlinear wave equation by Chewning [2] and Fattorini [4]. More recently, by applying Schauder's fixed point theorem, Zuazua [15, 16] has further examined local controllability for the nonlinear wave equation. Moreover, the reachability problem for thermoelastic plates has been considered by Lagnese [6].

For the semilinear plate equation, Lasiecka and Triggiani [7] have discussed the following boundary control problem

$$\begin{cases} y'' + \Delta^2 y + f(y) = 0 & \text{in } Q, \\ y(0) = y^0, \quad y'(0) = y^1 & \text{in } \Omega, \\ y = \phi_1, \quad \Delta y = \phi_2 & \text{on } \Sigma, \end{cases} \tag{1.3}$$

with the nonlinearity $f(y)$ satisfying the following assumptions: f' is absolutely continuous and for some constant c

$$|f'(y)| + |f''(y)| \le c \qquad \text{for a.e. } \ y \in I\!R. \tag{1.4}$$

Under certain additional conditions, with the help of the global implicit function theorem, they were able to show that the system is (globally) exactly controllable. Here (globally) exact controllability means that every initial state (and not just those in a neighborhood of 0) in a suitable Hilbert space can be driven to rest.

We note that, in this previous work, the nonlinearity $f(y)$ is assumed to be globally Lipschitz. The main goal of this paper is to consider the case when the nonlinearity f is super-linear. Thus, we make the following super-linear assumption on f.

(H) Assume that $f(y) \in W^{1,\infty}_{loc}(I\!R)$ and $f(0) = 0$, and assume that there exist constants $k > 0$ and $p > 1$ such that

$$|f'(y)| \le k \mid y \mid^{p-1}, \qquad y \in I\!R \tag{1.5}$$

with

$$1 < p < 2 \text{ for } 1 \le n \le 4 \text{ or } 1 < p \le 1 + \frac{4}{n} \text{ for } n \ge 5. \tag{1.6}$$

We can compare (1.6) with the condition on p introduced in [15]. In this paper, in order to study the controllability for the semilinear wave equation in the super-linear case, Zuazua made the following assumption on $f(y)$: $f(y)$ satisfies (1.5) with

$$1 < p \leq 2 \text{ for } n = 1 \text{ or } p < 1 + \frac{2}{n} \text{ for } n \geq 2. \tag{1.7}$$

It is clear that the range of p in (1.6) for the semilinear plate equation is larger than that of p in (1.7) for the semilinear wave equation. Moreover, the critical exponent $p = 1 + \dfrac{4}{n}$ for $n \geq 5$ is included in (1.6). Possibly the discussion of the critical exponent case is the most interesting part of this paper since there are some technical difficulties involved in this case.

We note that we could try to include the linear term μy in the nonlinear function $f(y)$. However, condition (1.5) applied to $\mu y + f(y)$ would require $\mu = 0$. In fact, the addition of the extra term μy is nontrivial and, in order to treat this case, we first had to derive some results for the plate equation with lower-order terms. The results have been presented in our paper [11].

In order to present the main result of this paper, we introduce some notation which was used in [9]. Let $x^0 \in I\!R^n$ and $\nu = (\nu_1, \nu_2, \cdots, \nu_n)$ be the unit normal of Γ pointing towards the exterior of Ω. Set

$$m(x) = x - x^0 = (x_k - x_k^0),$$

$$\Gamma(x^0) = \{x \in \Gamma \ : \ m(x) \cdot \nu(x) = m_k(x)\nu_k(x) > 0\},$$
$$\Gamma_*(x^0) = \Gamma - \Gamma(x^0) = \{x \in \Gamma \ : \ m(x) \cdot \nu(x) \leq 0\},$$
$$\Sigma(x^0) = \Gamma(x^0) \times (0, T),$$
$$\Sigma_*(x^0) = \Gamma_*(x^0) \times (0, T).$$

In this paper, $H^s(\Omega)$ denotes the usual Sobolev space and $\| \cdot \|_s$ denotes its norm for any $s \in I\!R$. For $s \geq 0$, $H_0^s(\Omega)$ denotes the completion of $C_0^\infty(\Omega)$ in $H^s(\Omega)$, where $C_0^\infty(\Omega)$ is the space of all infinitely differentiable functions with compact support in Ω. Let X be a Banach space. We denote by $C^k([0,T], X)$ the space of all k times continuously differentiable functions defined on $[0,T]$ with values in X, and write $C([0,T], X)$ for $C^0([0,T], X)$.

The main result of the paper is the following.

Theorem 1.2. *Let Ω be a bounded domain in $I\!R^n$ with a boundary Γ of class C^3. Let $T > 0$. Assume (H) holds. Further, if $n \geq 5$ and $p = 1 + \dfrac{4}{n}$, then suppose that $f'(y)$ is continuous on $I\!R$. Then, the system (1.1) is locally controllable in $L^2(\Omega) \times H^{-2}(\Omega)$. That is, there exists a neighbourhood ϑ of $(0,0)$ in $L^2(\Omega) \times H^{-2}(\Omega)$ such that for any $(y^0, y^1) \in \vartheta$ there exists a control $\phi \in L^2(\Sigma(x^0))$ such that the solution $y = y(x, t; \phi)$ of (1.1) satisfies (1.2).*

Note that, in the critical exponent case, we have had to assume greater regularity for f.

Also note that, in the super-linear case, we cannot, in general, expect to be able to prove global controllability. Indeed, for this case only local existence results are known for the differential equation, let alone the control problem.

In the next section, we will apply Schauder's fixed point theorem to prove Theorem 1.2 in the case where $1 < p < 2$ for $1 \leq n \leq 4$ or $1 < p < 1 + \frac{4}{n}$ for $n \geq 5$. However, Schauder's fixed point theorem can not be applied in the case where $p = 1 + \frac{4}{n}$ because the operator constructed below (see (2.12)) may not be compact. Instead we appeal to the inverse function theorem. For this, one needs to prove that the operator is Fréchet differentiable. In our view, the discussion of Fréchet differentiablity of the operator is of independent interest.

2. Proof of Main Result

Consider the operator $A = \Delta^2 : D(A) \subset L^2(\Omega) \to L^2(\Omega)$ with domain $D(A) = H^4(\Omega) \cap H_0^2(\Omega)$. A is a strictly positive self-adjoint operator on $L^2(\Omega)$. By the definition of the intermediate spaces $[H_0^2(\Omega), L^2(\Omega)]_\sigma$ for $\sigma \in [0,1]$ (see [10, Chap.1, Definition 2.1]), we have

$$[H_0^2(\Omega), L^2(\Omega)]_\sigma = D(A^{\frac{1-\sigma}{2}})$$

with the norm being that of the graph of $A^{\frac{1-\sigma}{2}}$

$$(\| u \|_0^2 + \| A^{\frac{1-\sigma}{2}} u \|_0^2)^{\frac{1}{2}},$$

which is equivalent to

$$\| A^{\frac{1-\sigma}{2}} u \|_0 . \tag{2.1}$$

Here $D(A^{\frac{1-\sigma}{2}})$ is the domain of $A^{\frac{1-\sigma}{2}}$.

On the other hand, by Theorem 11.6 of Chapter 1 of [10], we have

$$H_0^{2(1-\sigma)}(\Omega) = [H_0^2(\Omega), L^2(\Omega)]_\sigma$$

if $\sigma \neq \frac{1}{4}, \frac{3}{4}$. Thus the usual norm on $H_0^{2(1-\sigma)}(\Omega)$ is equivalent to the norm (2.1) if $\sigma \neq \frac{1}{4}, \frac{3}{4}$.

Similarly, the usual norm on $H_0^{-2\sigma}(\Omega)$ is equivalent to

$$\| A^{-\frac{\sigma}{2}} u \|_0$$

for $\sigma \in [0,1]$ with $\sigma \neq \frac{1}{4}, \frac{3}{4}$.

If $\sigma = \dfrac{1}{4}, \dfrac{3}{4}$, then set

$$[H_0^2(\Omega), L^2(\Omega)]_{\frac{1}{4}} = H_{00}^{\frac{3}{2}}(\Omega), \qquad [H_0^2(\Omega), L^2(\Omega)]_{\frac{3}{4}} = H_{00}^{\frac{1}{2}}(\Omega).$$

It is well known from Theorem 11.7 of Chapter 1 of [10] that the spaces $H_{00}^{\frac{1}{2}}(\Omega)$ and $H_{00}^{\frac{3}{2}}(\Omega)$ are strictly contained in $H_0^{\frac{1}{2}}(\Omega)$ and $H_0^{\frac{3}{2}}(\Omega)$ with strict- ly finer topologies, respectively.

For $0 \leq s \leq 2$, set

$$\mathcal{H}_s = \begin{cases} H_0^s(\Omega) \times H^{s-2}(\Omega), & \text{if } s \neq \frac{1}{2},\ \frac{3}{2}, \\ H_{00}^{\frac{1}{2}}(\Omega) \times (H_{00}^{\frac{1}{2}}(\Omega))', & \text{if } s = \frac{1}{2}, \\ H_{00}^{\frac{3}{2}}(\Omega) \times (H_{00}^{\frac{3}{2}}(\Omega))', & \text{if } s = \frac{3}{2}. \end{cases}$$

The above argument allows us to introduce the following energy scalar product on $\mathcal{H}_s$:

$$\frac{1}{2}[(A^{\frac{s}{4}}u, A^{\frac{s}{4}}y) + (A^{\frac{s-2}{4}}v, A^{\frac{s-2}{4}}z)], \qquad \forall (u,v),\ (y,z) \in \mathcal{H}_s, \tag{2.2}$$

which is equivalent to the usual one.

In order to use the theory of semigroups of linear bounded operators, we define an operator U for $0 \leq s \leq 2$ by

$$U = \begin{pmatrix} 0 & I \\ -A & 0 \end{pmatrix} \tag{2.3}$$

on the Hilbert space $\mathcal{H}_s$ with domain

$$D(U) = \begin{cases} (H^4(\Omega) \cap H_0^2(\Omega)) \times H_0^s(\Omega), & \text{if } s \neq \frac{1}{2}, \frac{3}{2}, \\ (H^4(\Omega) \cap H_0^2(\Omega)) \times H_{00}^{\frac{1}{2}}(\Omega), & \text{if } s = \frac{1}{2}, \\ (H^4(\Omega) \cap H_0^2(\Omega)) \times H_{00}^{\frac{3}{2}}(\Omega), & \text{if } s = \frac{3}{2}. \end{cases}$$

It is well known that U is the infinitesimal generator of a strongly continuous semigroup of contractions on $\mathcal{H}_s$.

We are now in a position to prove Theorem 1.2.

Proof of Theorem 1.2. Given initial conditions

$$(u^0, u^1) \in H_0^2(\Omega) \times L^2(\Omega), \tag{2.4}$$

we consider the problem

$$\begin{cases} u'' + \Delta^2 u + \mu u = 0 & \text{in } Q, \\ u(0) = u^0, \quad u'(0) = u^1 & \text{in } \Omega, \\ u = \dfrac{\partial u}{\partial \nu} = 0 & \text{on } \Sigma. \end{cases} \tag{2.5}$$

It is well known that (2.5) has a unique weak solution u with $\Delta u \in L^2(\Sigma)$ (see [9, Chap. 4] and [11]).

Using the solution u of (2.5), we then consider the problem

$$\begin{cases} y'' + \Delta^2 y + \mu y + f(y) = 0 & \text{in } Q, \\ y(T) = 0, \quad y'(T) = 0 & \text{in } \Omega, \\ y = 0 & \text{on } \Sigma, \\ \dfrac{\partial y}{\partial \nu} = \begin{cases} \Delta u & \text{on } \Sigma(x^0), \\ 0 & \text{on } \Sigma_*(x^0). \end{cases} \end{cases} \tag{2.6}$$

In order to solve problem (2.6), we write the solution y of (2.6) as

$$y = w + z,$$

where w and z are, repectively, the solutions of

$$\begin{cases} w'' + \Delta^2 w + \mu w = 0 & \text{in } Q, \\ w(T) = 0, \quad w'(T) = 0 & \text{in } \Omega, \\ w = 0 & \text{on } \Sigma, \\ \dfrac{\partial w}{\partial \nu} = \begin{cases} \Delta u & \text{on } \Sigma(x^0), \\ 0 & \text{on } \Sigma_*(x^0), \end{cases} \end{cases} \tag{2.7}$$

$$\begin{cases} z'' + \Delta^2 z + \mu z + f(w+z) = 0 & \text{in } Q, \\ z(T) = 0, \quad z'(T) = 0 & \text{in } \Omega, \\ z = \dfrac{\partial z}{\partial \nu} = 0 & \text{on } \Sigma. \end{cases} \tag{2.8}$$

Problem (2.7) has a unique weak solution w (see [9, Chap. 4] and [11]) with

$$w \in C([0,T]; L^2(\Omega)) \cap C([0,T]; H^{-2}(\Omega)).$$

Moreover, there exists $c > 0$ such that for every $t \in [0,T]$

$$\begin{aligned} \| w(t) \|_0 + \| w'(t) \|_{-2} &\le c \| \Delta u \|_{L^2(\Sigma(x^0))} \\ &\le c[\| u^0 \|_2 + \| u^1 \|_0]. \end{aligned} \tag{2.9}$$

By Lemma 2.2 below, we will see that there exist positive constants $0 \le s < 2$ and r such that for every

$$(u^0, u^1) \in \overline{B}_r(0) = \{(u^0, u^1) \in H_0^2(\Omega) \times L^2(\Omega) \ : \ \|(u^0, u^1)\|_{H_0^2(\Omega) \times L^2(\Omega)} \le r\},$$

problem (2.8) has a unique weak solution z with

$$(z, z') \in C([0,T]; \mathcal{H}_s). \tag{2.10}$$

Moreover, there exists a constant $c > 0$ independent of (u^0, u^1) such that for all $t \in [0,T]$

$$\| (z(t), z'(t)) \|_{\mathcal{H}_s} \le c[\| u^0 \|_2 + \| u^1 \|_0]^p. \tag{2.11}$$

We now define a nonlinear operator W by

$$\begin{aligned} W(u^0,u^1) &= (y'(0),-y(0)) \\ &= (w'(0),-w(0))+(z'(0),-z(0)) \\ &= \Lambda(u^0,u^1)+K(u^0,u^1). \end{aligned} \tag{2.12}$$

Given $(y^0,y^1)\in L^2(\Omega)\times H^{-2}(\Omega)$, we look at the problem

$$W(u^0,u^1)=(y^1,-y^0). \tag{2.13}$$

Once we have shown that there exists a neighbourhood ϑ of $(0,0)$ in $L^2(\Omega)\times H^{-2}(\Omega)$ such that for any $(y^0,y^1)\in\vartheta$ problem (2.13) has a solution, then the theorem is proved. We solve (2.13) in two cases.

Case (i): $1<p<2$ and $1\le n\le 4$ or $1<p<1+\dfrac{4}{n}$ and $n\ge 5$. In this case, we are going to apply Schauder's fixed point theorem.

Since the operator Λ is an isomorphism from $H_0^2(\Omega)\times L^2(\Omega)$ onto $H^{-2}(\Omega)\times L^2(\Omega)$ (see [11]), problem (2.13) is equivalent to

$$\begin{aligned} (u^0,u^1) &= -\Lambda^{-1}K(u^0,u^1)+\Lambda^{-1}(y^1,-y^0) \\ &= G(u^0,u^1). \end{aligned} \tag{2.14}$$

In this case, we can take $0<s<2$ in Lemma 2.1 below. Consequently, it follows from (2.10), (2.11), and the compact embedding theorem (see [10, p.99]) that the operator K is compact from $H_0^2(\Omega)\times L^2(\Omega)$ to $H^{-2}(\Omega)\times L^2(\Omega)$. Moreover, by Lemma 2.3 below, K is continuous. To apply Schauder's fixed point theorem, we must find $\tau\in(0,r]$ such that G maps $\overline{B}_\tau(0)$ into $\overline{B}_\tau(0)$. By (2.11), we deduce that there is a constant $c>0$ such that

$$\begin{aligned} \| G(u^0,u^1)\|_{H_0^2(\Omega)\times L^2(\Omega)} &\le \| -\Lambda^{-1}K(u^0,u^1)\|_{H_0^2(\Omega)\times L^2(\Omega)} \\ &\quad + \|\Lambda^{-1}(y^1,-y^0)\|_{H_0^2(\Omega)\times L^2(\Omega)} \\ &\le c\tau^p + \|\Lambda^{-1}(y^1,-y^0)\|_{H_0^2(\Omega)\times L^2(\Omega)}, \end{aligned}$$

for any $(u^0,u^1)\in\overline{B}_\tau(0)$. Thus, it is enough to choose $\tau\in(0,r]$ such that

$$c\tau^p + \|\Lambda^{-1}(y^1,-y^0)\|_{H_0^2(\Omega)\times L^2(\Omega)}\le\tau.$$

This is possible if we take

$$\|\Lambda^{-1}(y^1,-y^0)\|_{H_0^2(\Omega)\times L^2(\Omega)}\le \min\Big\{\frac{1}{(cp)^{\frac{1}{p-1}}}\Big(1-\frac{1}{p}\Big),\ |r-cr^p|\Big\}. \tag{2.15}$$

By Schauder's fixed point theorem, G has a fixed point for $(y^0,y^1)\in L^2(\Omega)\times H^{-2}(\Omega)$ satisfying (2.15), and consequently, equation (2.13) has a solution.

Case (ii): $n \geq 5$ and $p = 1 + \dfrac{4}{n}$, and $f'(y)$ is continuous on $I\!R$. In this case, Schauder's fixed point theorem can be no longer applied because the constant s of Lemma 2.1 below is equal to 0, and consequently, the operator K defined in (2.12) is no longer compact. Instead we appeal to the classical inverse function theorem (see [3, p.149]). For this, we first deduce from (2.11) that $K'(0,0) = 0$ since $p > 1$ and $f(0) = 0$, where $K'(0,0)$ denotes the Fréchet derivative of K at $(0,0)$. Therefore, $W'(0,0) = \Lambda$. In addition, by Lemma 2.4 below, K is continuously Fréchet differentiable on $B_r(0) \subset H_0^2(\Omega) \times L^2(\Omega)$. It therefore follows from the classical inverse function theorem that W is a local homeomorphism, i.e., there is a neighbourhood O of $(0,0)$ such that W is a homeomorphism onto the neighbourhood $\vartheta = W(O)$ of $(0,0) = W(0,0)$. Hence for any $(y^0, y^1) \in \vartheta$ problem (2.13) has a solution. Thus, the proof of Theorem 1.2 is complete provided we can prove the following four lemmas. $\square$

Lemma 2.1. *Suppose assumption (H) holds. Then there is a $0 \leq s < 2$ such that $f(y) : L^2(\Omega) \to H^{s-2}(\Omega)$ is locally Lipschitz continuous, that is, for every constant $c \geq 0$ there is a constant $l(c)$ such that*

$$\| f(y_1) - f(y_2) \|_{s-2} \leq l(c) \| y_1 - y_2 \|_0,$$

for all $y_1, y_2 \in L^2(\Omega)$ with $\| y_1 \|_0 \leq c$, $\| y_2 \|_0 \leq c$. Moreover, the constant s satisfies:

(i) if $1 < p < 2$ and $1 \leq n \leq 4$ or $1 < p < 1 + \dfrac{4}{n}$ and $n \geq 5$, then $0 < s < 2$;

(ii) if $p = 1 + \dfrac{4}{n}$ and $n \geq 5$, then $s = 0$.

Proof. Set

$$q = \frac{2}{2-p}.$$

It follows from (1.6) that

$$0 > \frac{n}{q} - \frac{n}{2} = \frac{n}{2}(1-p) \geq -2.$$

Set

$$\varepsilon = \frac{n}{2} - \frac{n}{q}, \qquad s = 2 - \varepsilon,$$

then s satisfies (i) and (ii). It follows from Sobolev's embedding theorem (see [1, p.218]) that $H_0^{\varepsilon}(\Omega)$ is continuously embeded into $L^q(\Omega)$. Consequently, $L^{\frac{2}{p}}(\Omega)$ is continuously embeded into $H^{-\varepsilon}(\Omega) = H^{s-2}(\Omega)$. It therefore follows from Hölder's inequality and assumption (H) that for any $y_1, y_2 \in L^2(\Omega)$

$$\begin{aligned} \| f(y_1) - f(y_2) \|_{s-2} &\leq c \| f(y_1) - f(y_2) \|_{0,\frac{2}{p}} \\ &\leq c \| (| y_1 |^{p-1} + | y_2 |^{p-1})(y_1 - y_2) \|_{0,\frac{2}{p}} \\ &\leq c \|| y_1 |^{p-1} + | y_2 |^{p-1}\|_{0,\frac{2}{p-1}} \| y_1 - y_2 \|_0 \\ &\leq c(\| y_1 \|_0^{p-1} + \| y_2 \|_0^{p-1}) \| y_1 - y_2 \|_0, \end{aligned}$$

where $\|\cdot\|_{0,q}$ denotes the norm of $L^q(\Omega)$. This implies the lemma. $\square$

Since problems (2.7) and (2.8) are time-reversible, we may consider the following problems instead of (2.7) and (2.8):

$$\begin{cases} w'' + \Delta^2 w + \mu w = 0 & \text{in } Q, \\ w(0) = 0, \quad w'(0) = 0 & \text{in } \Omega, \\ w = 0 & \text{on } \Sigma, \\ \dfrac{\partial w}{\partial \nu} = \begin{cases} \Delta u & \text{on } \Sigma(x^0), \\ 0 & \text{on } \Sigma_*(x^0), \end{cases} \end{cases} \tag{2.16}$$

$$\begin{cases} z'' + \Delta^2 z + \mu z + f(w+z) = 0 & \text{in } Q, \\ z(0) = 0, \quad z'(0) = 0 & \text{in } \Omega, \\ z = \dfrac{\partial z}{\partial \nu} = 0 & \text{on } \Sigma. \end{cases} \tag{2.17}$$

Lemma 2.2. *Suppose that (H) holds. Then there exist positive constants $0 \le s < 2$ and r such that for every*

$$(u^0, u^1) \in \overline{B}_r(0) = \{(u^0, u^1) \in H_0^2(\Omega) \times L^2(\Omega) \ : \ \|(u^0, u^1)\|_{H_0^2(\Omega)\times L^2(\Omega)} \le r\},$$

problem (2.17) has a unique weak solution z with

$$(z, z') \in C([0,T]; \mathcal{H}_s). \tag{2.18}$$

Moreover, there exists a constant $c > 0$, independent of (u^0, u^1), such that for all $t \in [0,T]$

$$\| (z(t), z'(t)) \|_{\mathcal{H}_s} \le c[\| u^0 \|_2 + \| u^1 \|_0]^p. \tag{2.19}$$

Proof. It follows from Lemma 2.1 and the theory of semigroups ([13, p.185]) that for each $(u^0, u^1) \in H_0^2(\Omega) \times L^2(\Omega)$ there exists some t_{max} depending on (u^0, u^1) such that problem (2.17) has a unique solution with

$$(z, z') \in C([0, t_{max}); \mathcal{H}_s).$$

Moreover, the following alternative holds: Either $t_{max} > T$ and (2.18) holds, or $t_{max} \le T$ and

$$\lim_{t \to t_{max}} \| (z(t), z'(t)) \|_{\mathcal{H}_s} = +\infty.$$

We are going to prove (2.18) and (2.19) hold for (u^0, u^1) small enough.

We define the energy of a solution of (2.17) by

$$E(z,t) = \frac{1}{2}[\| A^{\frac{s}{4}} z(t) \|_0^2 + \| A^{\frac{s-2}{4}} z'(t) \|_0^2]. \tag{2.20}$$

Multiplying (2.17) by $A^{\frac{s-2}{2}}z'$ and integrating over $Q_t = \Omega \times (0,t)$, it follows that (the following c's denoting various constants depending on T, μ, the operator A, and the constants k, p in (1.5))

$$
\begin{aligned}
& E(z,t) \\
& = -\int_{Q_t} f(w(t)+z(t))A^{\frac{s-2}{2}}z'(t)dxdt - \int_{Q_t} \mu z(t)A^{\frac{s-2}{2}}z'(t)dxdt \\
& \le \int_0^t \| A^{\frac{s-2}{4}} f(w(t)+z(t)) \|_0 \| A^{\frac{s-2}{4}} z'(t) \|_0 \, dt + c\int_0^t E(z,t)dt \\
& \text{(use the fact that } H^{s-2}(\Omega) \subset L^{\frac{2}{p}}(\Omega)) \\
& \le c\int_0^t \| f(w(t)+z(t)) \|_{0,\frac{2}{p}} \| A^{\frac{s-2}{4}} z'(t) \|_0 \, dt + c\int_0^t E(z,t)dt \\
& \text{(use hypothesis } (H)) \\
& \le c\int_0^t \| w(t)+z(t) \|_0^p \| A^{\frac{s-2}{4}} z'(t) \|_0 \, dt + c\int_0^t E(z,t)dt \\
& \le c \| w \|_{L^\infty(0,T;L^2(\Omega))}^{2p} + c\int_0^t E(z,t)dt \\
& \qquad + c\int_0^t (\| A^{\frac{s-2}{4}} z'(t) \|_0^2 + \| z(t) \|_0^p \| A^{\frac{s-2}{4}} z'(t) \|_0)dt \\
& \text{(use the property of fractional powers : } \| y(t) \|_0 \le c \| A^{\frac{s}{4}} y(t) \|_0) \\
& \le c \| (u^0,u^1) \|_{H_0^2(\Omega)\times L^2(\Omega)}^{2p} + c\int_0^t E(z,t)dt \\
& \qquad + c\int_0^t (\| A^{\frac{s-2}{4}} z'(t) \|_0^2 + \| A^{\frac{s}{4}} z(t) \|_0^p \| A^{\frac{s-2}{4}} z'(t) \|_0)dt \\
& \le d + c\int_0^t [E(z,t) + E^{\frac{1+p}{2}}(z,t)]dt,
\end{aligned}
\tag{2.21}
$$

where

$$d = c \| (u^0,u^1) \|_{H_0^2(\Omega)\times L^2(\Omega)}^{2p} .$$

On the other hand, the solution of the initial value problem

$$\begin{cases} u' = c(u + u^{\frac{1+p}{2}}), \\ u(0) = d, \end{cases}$$

is

$$u = \frac{de^{ct}}{[1 + d^{\frac{p-1}{2}} - d^{\frac{p-1}{2}} \exp(\frac{1}{2}c(p-1)t)]^{\frac{2}{p-1}}}.$$

It therefore follows from Corollary 6.5 of [5, p.35] that

$$
\begin{aligned}
E(z,t) & \le \frac{de^{ct}}{[1 + d^{\frac{p-1}{2}} - d^{\frac{p-1}{2}} \exp(\frac{1}{2}c(p-1)t)]^{\frac{2}{p-1}}} \\
& \le 2de^{ct},
\end{aligned}
$$

for $0 \leq t \leq T$ if

$$d = c \parallel (u^0, u^1) \parallel^{2p}_{H_0^2(\Omega)\times L^2(\Omega)} < \frac{1}{2[\exp(\frac{c(p-1)T}{2}) - 1]^{\frac{p-1}{2}}}.$$

Thus we have proved (2.18) and (2.19). □

In what follows, the c's denote various constants depending on T, μ, the operator A, the constants k, p in (1.5), and the constant r in Lemma 2.2. In addition, since we are considering problems (2.16) and (2.17), the operator K defined in (2.12) is now defined by

$$K(u^0, u^1) = (z'(T), -z(T)).$$

Lemma 2.3. *Suppose that (H) holds. Then the operator K defined in (2.12) is Lipschitz continuous from $B_r(0) \subset H_0^2(\Omega) \times L^2(\Omega)$ to $H^{s-2}(\Omega) \times H^s(\Omega)$, where r is the constant obtained in Lemma 2.2.*

Proof. Given $\xi_1 = (u_1^0, u_1^1)$ and $\xi_2 = (u_2^0, u_2^1) \in B_r(0) \subset H_0^2(\Omega) \times L^2(\Omega)$, by Lemma 2.2, problem (2.17) has unique solutions z_1, z_2, respectively. Let w_1, w_2 be the solutions of (2.16) corresponding to ξ_1, ξ_2. By (2.9) and (2.19), we have

$$\|w_i(t)\|_0 \leq cr, \quad \|z_i(t)\|_0 \leq cr^p, \qquad i = 1, 2, \ \forall t \in [0, T]. \tag{2.22}$$

From (2.17) it follows that

$$\begin{cases} (z_1 - z_2)'' + \Delta^2(z_1 - z_2) + \mu(z_1 - z_2) & \\ \qquad +f(w_1 + z_1) - f(w_2 + z_2) = 0 & \text{in } Q, \\ z_1(0) - z_2(0) = 0, \quad z_1'(0) - z_2'(0) = 0 & \text{in } \Omega, \\ z_1 - z_2 = \dfrac{\partial(z_1 - z_2)}{\partial\nu} = 0 & \text{on } \Sigma. \end{cases} \tag{2.23}$$

As in the proof of (2.21), multiplying (2.23) by $A^{\frac{s-2}{2}}(z_1 - z_2)'$ and integrating over $Q_t = \Omega \times (0, t)$, it follows that

$$\begin{aligned} & E(z_1 - z_2, t) \\ & \leq c \int_0^t (\|w_1(t) + z_1(t)\|_0^{p-1} + \|w_2(t) + z_2(t)\|_0^{p-1}) \\ & \qquad \times (\|w_2(t) - w_1(t)\|_0 + \|z_2(t) - z_1(t)\|_0) \parallel A^{\frac{s-2}{4}}(z_1 - z_2)'(t) \parallel_0 dt \\ & \qquad + c \int_0^t E(z_1 - z_2, t) dt \\ & \text{(use (2.22))} \\ & \leq c \int_0^t (r + r^p)^{p-1} (\|w_2(t) - w_1(t)\|_0 + \|z_2(t) - z_1(t)\|_0) \\ & \qquad \times \parallel A^{\frac{s-2}{4}}(z_1 - z_2)'(t) \parallel_0 dt + c \int_0^t E(z_1 - z_2, t) dt \\ & \leq c \parallel w_2 - w_1 \parallel^2_{L^\infty(0,T;L^2(\Omega))} + c \int_0^t E(z_1 - z_2, t) dt. \end{aligned}$$

It therefore follows from Gronwall's inequality (see [5, p.36]) that

$$
\begin{aligned}
E(z_1 - z_2, t) &\le ce^{ct} \parallel w_2 - w_1 \parallel^2_{L^\infty(0,T;L^2(\Omega))} \\
&\le ce^{ct} \parallel \xi_2 - \xi_1 \parallel^2_{H_0^2(\Omega)\times L^2(\Omega)} .
\end{aligned}
\tag{2.24}
$$

This implies that K is Lipschitz continuous on $B_r(0)$. □

Lemma 2.4. *Suppose that (H) holds. If $f'(y)$ is continuous on $\mathbb{R}$, then the operator $K: \ B_r(0) \subset H_0^2(\Omega) \times L^2(\Omega) \to H^{s-2}(\Omega) \times H^s(\Omega)$ defined in (2.12) is continuously Fréchet differentiable on $B_r(0)$.*

Proof. Let $\xi = (u^0, u^1) \in B_r(0) \subset H_0^2(\Omega) \times L^2(\Omega)$ and $\eta = (v^0, v^1) \in H_0^2(\Omega) \times L^2(\Omega)$. Let $w(\xi)$, $w(\eta)$ be the solutions of (2.16) corresponding to ξ, η, respectively, and $z(\xi)$ the solution of (2.17) corresponding to ξ. Consider the following linearized problem

$$
\begin{cases}
\psi'' + \Delta^2\psi + \mu\psi + f'(w(\xi) + z(\xi))(w(\eta) + \psi) = 0 & \text{in } Q, \\
\psi(0) = 0, \quad \psi'(0) = 0 & \text{in } \Omega, \\
\psi = \dfrac{\partial \psi}{\partial \nu} = 0 & \text{on } \Sigma,
\end{cases}
\tag{2.25}
$$

which has a unique solution $\psi = \psi(t, x; \xi, \eta)$ with

$$
(\psi, \psi') \in C([0,T], \mathcal{H}_s).
$$

Define the linear operator $L(\xi)$ by

$$
L(\xi)\eta = (\psi'(T; \xi, \eta), -\psi(T; \xi, \eta)), \quad \forall \eta = (u^0, u^1) \in H_0^2(\Omega) \times L^2(\Omega). \tag{2.26}
$$

We are going to prove that

$$
K'(\xi) = L(\xi), \qquad \forall \xi \in B_r(0) \tag{2.27}
$$

We first prove that $L(\xi)$ is a bounded operator from $H_0^2(\Omega)\times L^2(\Omega)$ to $H^{s-2}(\Omega)\times H^s(\Omega)$ for every $\xi \in B_r(0)$, and continuous with respect to ξ on $B_r(0)$.

As in the proof of (2.21), we can obtain

$$
\begin{aligned}
&E(\psi(\eta), t) \\
&\le c \int_0^t (\|w(t,\xi)\|_0^{p-1} + \|z(t,\xi)\|_0^{p-1}) \|(w(t,\eta) + \psi(t,\eta))\|_0 \\
&\qquad \times \parallel A^{\frac{s-2}{4}} \psi'(t,\eta) \parallel_0 dt + c\int_0^t E(\psi(\eta), t) dt \qquad \text{(use (2.22))} \\
&\le c(r^{p-1} + r^{p(p-1)}) \int_0^t \|(w(t,\eta) + \psi(t,\eta))\|_0 \parallel A^{\frac{s-2}{4}} \psi'(t,\eta) \parallel_0 dt \\
&\qquad + c\int_0^t E(\psi(\eta), t) dt \\
&\le c\int_0^T \|w(t,\eta)\|_0^2 dt + c\int_0^t E(\psi(\eta), t) dt.
\end{aligned}
$$

It therefore follows from Gronwall's inequality that

$$
\begin{aligned}
E(\psi(\eta),t) &\le ce^{ct} \parallel w(\eta) \parallel^2_{L^\infty(0,T;L^2(\Omega))} \\
&\le ce^{ct} \parallel \eta \parallel^2_{H_0^2(\Omega)\times L^2(\Omega)} .
\end{aligned}
\tag{2.28}
$$

This implies that $L(\xi)$ is bounded.

Suppose $\xi_1,\ \xi_2 \in B_r(0)$. Let $\psi_1,\ \psi_2$ be the solutions of (2.25) with ξ replaced by $\xi_1,\ \xi_2$, respectively. Then we have

$$
\begin{cases}
(\psi_1-\psi_2)''+\Delta^2(\psi_1-\psi_2)+\mu(\psi_1-\psi_2) & \\
\quad +f'(w(\xi_1)+z(\xi_1))(w(\eta)+\psi_1) & \\
\qquad -f'(w(\xi_2)+z(\xi_2))(w(\eta)+\psi_2)=0 & \text{in } Q, \\
\psi_1(0)-\psi_2(0)=0, \quad \psi_1'(0)-\psi_2'(0)=0 & \text{in } \Omega, \\
\psi_1-\psi_2=\dfrac{\partial(\psi_1-\psi_2)}{\partial\nu}=0 & \text{on } \Sigma.
\end{cases}
\tag{2.29}
$$

Set

$$\sigma(t,\xi_1,\xi_2)=f'(w(\xi_2)+z(\xi_2))-f'(w(\xi_1)+z(\xi_1)).$$

As before, we can obtain

$$
\begin{aligned}
&E(\psi_1(\eta)-\psi_2(\eta),t) \\
&= \int_{Q_t} f'(w(t,\xi_1)+z(t,\xi_1))(\psi_2(t,\eta)-\psi_1(t,\eta)) \\
&\quad \times A^{\frac{s-2}{2}}(\psi_1(\eta)-\psi_2(\eta))'(t)dxdt \\
&\quad + \int_{Q_t} \sigma(t,\xi_1,\xi_2)(w(t,\eta)+\psi_2(t,\eta))A^{\frac{s-2}{2}}(\psi_1(\eta)-\psi_2(\eta))'(t)dxdt \\
&\quad - \int_{Q_t} \mu(\psi_1(t,\eta)-\psi_2(t,\eta))A^{\frac{s-2}{2}}(\psi_1(\eta)-\psi_2(\eta))'(t)dxdt \\
&\le c\int_0^t \|w(t,\xi_1)+z(t,\xi_1)\|_0^{p-1}\|\psi_2(t,\eta)-\psi_1(t,\eta)\|_0 \\
&\quad \times \|A^{\frac{s-2}{4}}(\psi_1(\eta)-\psi_2(\eta))'(t)\|_0 dt \\
&\quad + c\int_0^t \|\sigma(t,\xi_1,\xi_2)\|_{0,\frac{2}{p-1}}\|w(t,\eta)+\psi_2(t,\eta)\|_0 \\
&\quad \times \|A^{\frac{s-2}{4}}(\psi_1(\eta)-\psi_2(\eta))'(t)\|_0 dt \\
&\quad + c\int_0^t E(\psi_1(\eta)-\psi_2(\eta),t)dt \quad (\text{use } \|w(t,\xi_1)+z(t,\xi_1)\|_0 \le c(r+r^p)) \\
&\le c\int_0^T \|\sigma(t,\xi_1,\xi_2)\|^2_{0,\frac{2}{p-1}}\|w(t,\eta)+\psi_2(t,\eta)\|_0^2 dt \\
&\quad + c\int_0^t E(\psi_1(\eta)-\psi_2(\eta),t)dt.
\end{aligned}
$$

It therefore follows from (2.9), (2.28), and Gronwall's inequality that

$$
\begin{aligned}
& E(\psi_1(\eta) - \psi_2(\eta), t) \\
& \leq ce^{ct} \parallel w(\eta) + \psi_2(\eta) \parallel^2_{L^\infty(0,T;L^2(\Omega))} \int_0^T \|\sigma(t,\xi_1,\xi_2)\|^2_{0,\frac{2}{p-1}} dt \\
& \leq ce^{ct} \parallel \eta \parallel^2_{H_0^2(\Omega)\times L^2(\Omega)} \int_0^T \|\sigma(t,\xi_1,\xi_2)\|^2_{0,\frac{2}{p-1}} dt.
\end{aligned}
\tag{2.30}
$$

This implies that

$$
\|L(\xi_1) - L(\xi_2)\|^2 \leq c \int_0^T \|\sigma(t,\xi_1,\xi_2)\|^2_{0,\frac{2}{p-1}} dt. \tag{2.31}
$$

We now show that

$$
\lim_{\xi_1 \to \xi_2} \|\sigma(t,\xi_1,\xi_2)\|_{0,\frac{2}{p-1}} = 0, \quad \forall t \in [0,T]. \tag{2.32}
$$

Let $\{\xi_{1n}\}$ be any sequence converging to ξ_2. Then by (2.9) and (2.24) we have $w(\xi_{1n}) \to w(\xi_2)$ and $z(\xi_{1n}) \to z(\xi_2)$ in $L^2(\Omega)$ as $n \to \infty$ for all $t \in [0,T]$. Hence, there is a subsequence $\{\xi_{1n_i}\}$ such that $w(\xi_{1n_i}) \to w(\xi_2)$ and $z(\xi_{1n_i}) \to z(\xi_2)$ a.e. on Ω as $n \to \infty$ for all $t \in [0,T]$. By the continuity of $f'(y)$, we have $\sigma(t,\xi_{1n_i},\xi_2) \to 0$ a.e. on Ω as $n \to \infty$ for all $t \in [0,T]$. On the other hand, it follows from (1.5), (2.9) and (2.19) that

$$
\|\sigma(t,\xi_{1n_i},\xi_2)\|_{0,\frac{2}{p-1}} \leq c(r^{p-1} + r^{p(p-1)}).
$$

It therefore follows from Lebesgue's dominated convergence theorem that

$$
\lim_{i\to\infty} \|\sigma(t,\xi_{1n_i},\xi_2)\|_{0,\frac{2}{p-1}} = 0, \qquad \forall t \in [0,T].
$$

This shows that (2.32) holds. Hence it follows from (2.31) and (2.32) that $L(\xi)$ is continuous with respect to ξ on $B_r(0)$.

Finally, we prove (2.27). Set

$$
\begin{aligned}
\theta &= z(\xi+\eta) - z(\xi) - \psi(\xi,\eta), \\
\omega(\eta) &= f'(w(\xi) + z(\xi)) \\
&\quad - f'(\alpha(w(\xi)+z(\xi)) + (1-\alpha)(w(\xi+\eta) + z(\xi+\eta))), \\
\delta(\eta) &= f'(\alpha(w(\xi)+z(\xi)) + (1-\alpha)(w(\xi+\eta) + z(\xi+\eta))),
\end{aligned}
$$

where $0 < \alpha < 1$. As before, we can obtain

$$
\begin{aligned}
& E(\theta,t) \\
& = -\int_{Q_t} [f(w(t,\xi+\eta) + z(t,\xi+\eta)) - f(w(t,\xi) + z(t,\xi))] A^{\frac{s-2}{2}} \theta'(t) dxdt \\
& \quad + \int_{Q_t} f'(w(t,\xi) + z(t,\xi))(w(t,\eta) + \psi(t,\eta)) A^{\frac{s-2}{2}} \theta'(t) dxdt \\
& \quad - \int_{Q_t} \mu\theta(t) A^{\frac{s-2}{2}} \theta'(t) dxdt
\end{aligned}
$$

$$
\begin{aligned}
&= -\int_{Q_t} f'(\alpha(w(t,\xi)+z(t,\xi))+(1-\alpha)(w(t,\xi+\eta)+z(t,\xi+\eta))) \\
&\quad \times (w(t,\eta)+z(t,\xi+\eta)-z(t,\xi))A^{\frac{s-2}{2}}\theta'(t)dxdt \\
&\quad + \int_{Q_t} f'(w(t,\xi)+z(t,\xi))(w(t,\eta)+\psi(t,\eta))A^{\frac{s-2}{2}}\theta'(t)dxdt \\
&\quad + c\int_0^t E(\theta,t)dt \\
&= \int_{Q_t} \omega(t,\eta)w(t,\eta)A^{\frac{s-2}{2}}\theta'(t)dxdt - \int_{Q_t} \delta(t,\eta)\theta(t)A^{\frac{s-2}{2}}\theta'(t)dxdt \\
&\quad + \int_{Q_t} \omega(t,\eta)\psi(t,\eta)A^{\frac{s-2}{2}}\theta'(t)dxdt + c\int_0^t E(\theta,t)dt \\
&\le \int_0^t [\|\omega(t,\eta)\|_{0,\frac{2}{p-1}}\|w(t,\eta)\|_0 + \|\delta(t,\eta)\|_{0,\frac{2}{p-1}}\|\theta(t)\|_0]\|A^{\frac{s-2}{4}}\theta'(t)\|_0 dt \\
&\quad + \int_0^t [\|\omega(t,\eta)\|_{0,\frac{2}{p-1}}\|\psi(t,\eta)\|_0]\|A^{\frac{s-2}{4}}\theta'(t)\|_0 dt \\
&\quad + c\int_0^t E(\theta,t)dt \qquad (\text{use } \|\delta(t,\eta)\|_{0,\frac{2}{p-1}} \le c(r+r^p)^{p-1}) \\
&\le \int_0^T [\|\omega(t,\eta)\|^2_{0,\frac{2}{p-1}}\|w(t,\eta)\|^2_0 + \|\omega(t,\eta)\|^2_{0,\frac{2}{p-1}}\|\psi(t,\eta)\|^2_0]dt \\
&\quad + c\int_0^t E(\theta,t)dt.
\end{aligned}
$$

It therefore follows from (2.9), (2.28) and Gronwall's inequality that

$$
\begin{aligned}
&E(\theta,t) \\
&\le ce^{ct}\int_0^T [\|\omega(t,\eta)\|^2_{0,\frac{2}{p-1}}\|w(t,\eta)\|^2_0 + \|\omega(t,\eta)\|^2_{0,\frac{2}{p-1}}\|\psi(t,\eta)\|^2_0]dt \\
&\le ce^{ct} \parallel \eta \parallel^2_{H^2_0(\Omega)\times L^2(\Omega)} \int_0^T \|\omega(t,\eta)\|^2_{0,\frac{2}{p-1}}dt.
\end{aligned} \tag{2.33}
$$

As in the proof of (2.32), we can show that

$$
\lim_{\eta\to 0} \|\omega(t,\eta)\|_{0,\frac{2}{p-1}} = 0, \quad \forall t\in[0,T]. \tag{2.34}
$$

Thus (2.27) follows from (2.33) and (2.34). □

Remark 2.5. Another approach to proving Lemma 2.4 might be to use the following variation of parameters formula for problem (2.17):

$$
\Psi(t,\xi) = \int_0^t S(t-\tau)F(\Psi(\tau,\xi))d\tau,
$$

where $S(t)$ is the strongly continuous semigroup generated by U defined in (2.3) on $\mathcal{H}_s$, and

$$\xi = (u^0, u^1), \quad \Psi(t,\xi) = \begin{pmatrix} z(t,\xi) \\ z'(t,\xi) \end{pmatrix},$$

$$F(\Psi(t,\xi)) = \begin{pmatrix} 0 \\ \mu z(t,\xi) + f(z(t,\xi) + w(t,\xi)) \end{pmatrix}.$$

However, this method requires that $F'(\Psi)$ be locally Lipschitz. This requirement is not satisfied under the assumptions of Lemma 2.4. In any case, the proof of Lemma 2.4 gives a concrete expression for the Fréchet derivative of K. This can not be found with the variation of parameters formula.

Remark 2.6. The above method can be applied to the following control problem:

$$\begin{cases} y'' + \Delta^2 y + \mu y + f(y) = 0 & \text{in } Q, \\ y(0) = y^0, \quad y'(0) = y^1 & \text{in } \Omega, \\ y = 0, \quad \Delta u = \phi \chi_{\Sigma_0} & \text{on } \Sigma, \end{cases}$$

provided we can solve the corresponding problem of exact controllability for the plate equation with the lower-order term μy (and $f = 0$).

ACKNOWLEDGMENTS

The auther is deeply indebted to his supervisor Dr. Graham H. Williams for his guidance in the preparation of this paper. He is also grateful for the financial support from the Overseas Postgraduate Research Scholarship from Australian Government, the University of Wollongong Postgraduate Research Award, and the Department of Mathematics Analysis Research Group Scholarship from the Faculty of Informatics at the University of Wollongong. And he would like to thank the anonymous referee for bringing the paper [6] to his attention.

REFERENCES

[1] R. Adams, Sobolev Spaces, Academic Press, New York, 1975.

[2] W.C. Chewning, Controllability of the nonlinear wave equation in several space variables, *SIAM J. Control*, 14 (1976), 19-25.

[3] K. Deimling, Nonlinear Functional Analysis, Springer-Verlag, Berlin, 1985.

[4] H.O. Fattorini, Local controllability of a nonlinear wave equation, *Math. Systems Theory,* 9 (1975), 30-45.

[5] J.K. Hale, Ordinary Differential Equations, Wiley-interscience, New York, 1969.

[6] J. Lagnese, The reachability problem for thermoelastic plates, *Arch. Rational Mech. Anal.*, 112 (1990), 223-267.

[7] I. Lasiecka and R. Triggiani, Exact controllability of semilinear abstract systems with application to wave and plates boundary control problems, *Appl. Math. Optim.*, 23 (1991), 109-154.

[8] E.B. Lee and L. Markus, Foundation of Optimal Control Theory, John Wiley & Sons, Inc., New York, 1967.

[9] J.L. Lions, Contrôlabilité Exacte Perturbations et Stabilisation de Systè- mes Distribués, Tome 1, Contrôlabilité Exacte, Masson, Paris, 1988.

[10] J.L. Lions and E. Magenes, Non-homogeneous boundary value problems and applications, Vol. I and II, Springer-Verlag, New York, 1972.

[11] W.J. Liu and G.H. Williams, Exact controllability for problems of transmission for the plate equation with lower-order terms, *Quart. Appl. Math.,* to appear.

[12] L. Markus, Controllability for nonlinear processes, *SIAM J. Control,* 3 (1965), 78-90.

[13] A. Pazy, Semigroup of Linear Operators and Applications to Partial Differential Equations, Springer-Verlag, New York, 1983.

[14] G. Da Prato, Some results on controllability for semilinear equations, *in* Nonlinear Analysis (A. Ambrosetti and A. Marino, Ed.), pp.161-172, Quaderni, Scuola Normale Superiore, Pisa, 1991.

[15] E. Zuazua, Exact controllability for the semilinear wave equation, *J. Math. Pure Appl.,* 69 (1990), 1-31.

[16] E. Zuazua, Exact boundary controllability for the semilinear wave equation, *in* Nonlinear Partial Differential Equations and their Applications (H.O. Fattorini, Ed.), Collège de France Seminar, Vol X (Paris, 1987-1988), pp.357-391, Pitman Research Notes in Mathematics Series 220, Longman Science and Technology, Harlow, 1991.