Impedance Measurement of a Thermoacoustic Engine Constructed from a Helmholtz Resonator

by

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Abstract

A thermoacoustic engine was constructed from a 5-Liter Helmholtz flask by inserting a ceramic stack into the lower portion of the flask's neck. A cold heat exchanger and hot heat exchanger located on opposing faces of the stack provided very large temperature differences across the stack, and if the temperature difference was great enough then the system would produce sound. A previous experiment investigated how the resistance in the system changed as the temperature difference increased by observing changes in the quality factor; however, due to the nature of the measurements, it was not possible to observe how the imaginary part of the resistance changed as the temperature difference increased. This thesis focuses on how impedance measurements at the opening of the flask's neck might be analyzed in order to determine the frequency and temperature difference at which this system would reach onset and to see if this calculation for the onset temperature difference corresponds to the experimentally observed onset for the engine at 207°C from the prior experiment. This thesis will also focus on the behavior of the incident sound wave as the temperature difference across the engine's stack increases.

Contents

Acknowledgements

I am extremely grateful for the support of my research advisor and mentor, Dr. William Slaton, whose enthusiasm and dedication inspired me to explore thermoacoustics.

I am also thankful for the encouragement that I received from the faculty, staff, and my peers from the Department of Physics and Astronomy.

1 Theory

1.1 Helmholtz Resonators

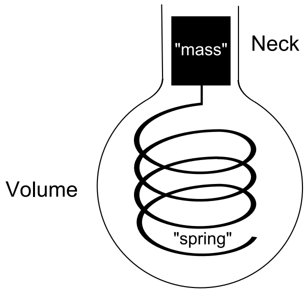


Figure 1: The behavior of air inside a Helmholtz resonator.

When a person blows air into a bottle, the air blown into the bottle causes the air inside the bottle to oscillate. If the person blows too hard or too softly, the air inside the bottle oscillates, but the amplitude of the oscillation is small. However, when a person blows into a bottle "just right," the amplitude becomes increasingly large, and the bottle will produce sound.

A bottle is a special kind of resonator known as a Helmholtz resonator. A Helmholtz resonator consists of a hollow neck attached to an empty volume. The air inside a Helmholtz resonator experiences pressure oscillations towards the bottom of the flask and velocity oscillations towards the opening.[1] For example, consider a mass bouncing on a spring. The air oscillating in the neck is comparable to the mass, and the air inside the cavity is comparable to the spring (see Figure 1). If the mass experiences friction, then its motion will become damped due to this added resistance. The air oscillating in the neck experiences friction due to its contact with the walls of the neck. The air inside a Helmholtz flask with a large resistance will experience a very weak response when driven at its natural frequency whereas a flask with little or no resistance will experience a very strong response.

1.2 Thermoacoustic Engines

Thermoacoustic engines are formally defined as heat engines "that exploit gas inertia, compliance, and resistance to create passive acoustical-phasing mechanisms."[2] A thermoacoustic engine is based upon the same principles as a thermodynamic engine. When heat is put into a thermoacoustic engine, the engine produces work in the form of sound with an amount of waste heat rejected into the cooler environment. However, unlike a traditional heat engine, a thermoacoustic engine containes no moving parts, thus making it a desirable alternative to tradition engines due to its high reliability and low cost.[3]

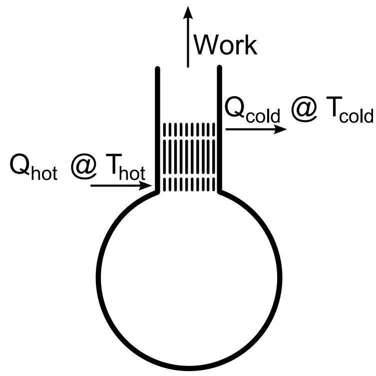


Figure 2: Thermoacoustic engine constucted from a Helmholtz resonator.

Consider a thermoacoustic engine made by inserting a porous ceramic stack into a Helmholtz flask as in Figure 2. Adding a porous ceramic material into a Helmholtz flask increases the system's resistance due to the friction between the oscillating air and the walls of the pores. However, the resistance of this system can be decreased by applying a temperature difference across the stack. If the system's resistance is eliminated, then the system produces work in the form of sound.

In order for the engine to work properly, the stack must be placed such that the air inside the stack can experience both velocity and pressure oscillations. It is possible to determine the proper location of the stack inside the flask by measuring the changes in pressure that occur in the flask as it is driven by an external source and inserting those pressure changes into Euler's equation, which states:

$$\rho_o \frac{\partial \vec{u}}{\partial t} = -\vec{\nabla} p \tag{1}$$

Rearranging Euler's equation to find the magnitude of the acoustic velocity gives:

$$u_{AC} = \left| \frac{1}{\rho_o \omega_o} \frac{\Delta P_o}{\Delta x} \right| \tag{2}$$

where u_{AC} is the acoustic velocity, ρ_o is the ambient air density, ω_o is the angular frequency, ΔP_o is the change in pressure, and Δx is the distance over which the pressure change occurs.[1] Therefore, if the pressure varies with position within a certain region of the flask, then the air contained in this volume will experience mainly velocity oscillations. If the pressure does not significantly vary with position within a certain region of the flask, then the air contained in this volume will experience mainly pressure oscillations. If there is no temperature difference across the stack and the stack is placed in a region that has both velocity and pressure oscillations, then a parcel of gas located within the stack will oscillate back and forth due to the velocity changes and also expand and contract as it travels due to the pressure changes.

If heat is applied to one end of the stack, then the parcel will now transfer this added heat energy down the stack. If the parcel is cooler than the hot end of the stack, it will absorb heat energy and thermally expand. This expansion of the parcel does net work on its environment. As it travels down the stack, the gas parcel will transfer its heat energy to the cooler stack, contract, and will repeat the process for as long as heat is provided into the system. The motion of this gas parcel is shown in Figure 3. This gas parcel along with its neighbors will act as a bucket brigade, transferring the heat energy from the hot end to the cooler end of the stack. The

[1]A simplified derivation of Equation 2 starting from Newton's Second Law can be seen in Appendix A.

net work of the gas parcels will provide the natural oscillation with acoustic power, and if the temperature difference across the stack is great enough, the engine will self-resonate or produce sound.

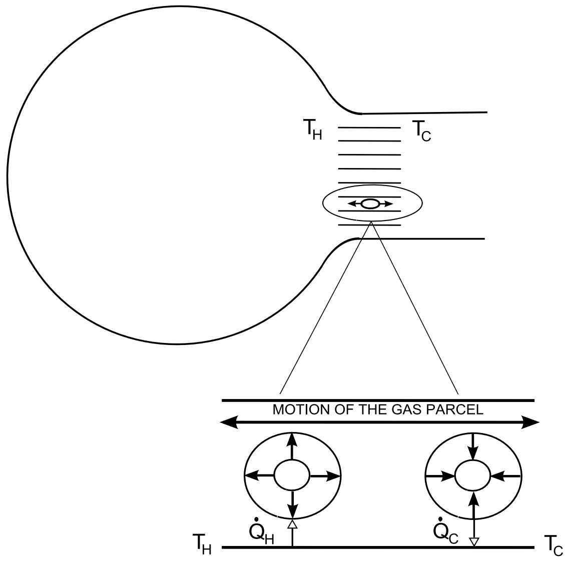


Figure 3: Motion of Gas Parcel

1.3 Experimental Results from a Prior Investigation

In a prior experiment, a thermoacoustic engine was constructed from a 5-Liter Helmholtz resonator, that had a neck with a cross-sectional area of 2.03×10^{-3} m^2 and and effective length of 8.90×10^{-2} m. From the resonance frequency equation for a Helmholtz resonator,

$$\nu_o = \frac{c}{2\pi}\sqrt{\frac{S}{L'V}} \tag{3}$$

where ν_o is the resonance frequency, c is the speed of sound, S is the cross-sectional area, L' is the effective length, and V is the volume, the resonance frequency for this particular flask was determined to be 97 Hertz. This calculation would be later supported by an experimental measurement.

From Newton's Second Law, the mass of air moving within the neck of the flask when driven by an external sinusoidal driving force can be represented by the equation:

$$m\frac{d^2x}{dt^2} + R\frac{dx}{dt} + kx = SPe^{i\omega t} \tag{4}$$

where m is the mass of the air, R is the resistance, k is the spring constant, and $SPe^{i\omega t}$ is the sinusoidal driving force. One assumes a particular solution of the form:

$$x_p(t) = xe^{i\omega t} \tag{5}$$

Inserting this solution into Equation 4 and solving for the magnitude of the displacement yields:

$$|x| = \frac{SP}{\frac{\nu_0\nu}{Q}\sqrt{1 + (\frac{\nu_0}{\nu})^2 Q^2[1 - (\frac{\nu}{\nu_0})^2]^2}} \tag{6}$$

In this equation, the resonance frequency, ν_o, and the quality factor. Q, are given by:

$$\omega_o = \sqrt{\frac{k}{m}} \tag{7}$$

and

$$Q = \frac{\omega_0 m}{R} \tag{8}$$

At resonance, or when ν=ν_o, Equation 6 can be written as:

$$x_{max} = \frac{SPQ}{\omega_\circ^2} = \frac{SPm}{\omega_\circ R} \tag{9}$$

Inserting the value for x_{max} into Equation 6 gives:

$$\frac{x}{x_{max}} = \frac{1}{\sqrt{1 + \left(\frac{f_0}{f}\right)^2 Q^2 \left[1 - \left(\frac{f}{f_0}\right)^2\right]^2}} \tag{10}$$

Equation 6 is the general solution for a simple harmonic oscillator. From the adiabatic equation of state, the displacement of the mass, x, can be written in terms of the pressure:

$$x = \frac{V_o}{\rho_o c^2 S} P \tag{11}$$

Therefore, Equation 6 can be written in terms of the amplitude of the pressure within the flask, A, to the maximum amplitude at resonance, A_{max}:

$$\frac{A}{A_{max}} = \frac{1}{\sqrt{1 + \left(\frac{f_0}{f}\right)^2 Q^2 \left[1 - \left(\frac{f}{f_0}\right)^2\right]^2}} \tag{12}$$

A thermoacoustic engine was created from the Helmholtz flask by inserting a porous ceramic stack[2] into the bottom portions of the flask's neck. Measurements of the pressure within the volume of the thermoacoustic engine were taken as it was swept over a range of frequencies. This setup can be viewed in Figure 4.

The measurements were then plotted and fit to Equation 12. From the fit, the resonance frequency and the quality factor of the system were determined. A total of eight different measurements for the quality factor were made for eight increasing temperature differences across the stack, and the quality factor was shown to increase as the temperature difference increased. At a temperature difference of 207°C, the thermoacoustic engine reached onset. A summary of the results from this experiment can be viewed in Appendix B.

[2]The ceramic stack used in this experiment is identical to the stack described in Section 2.

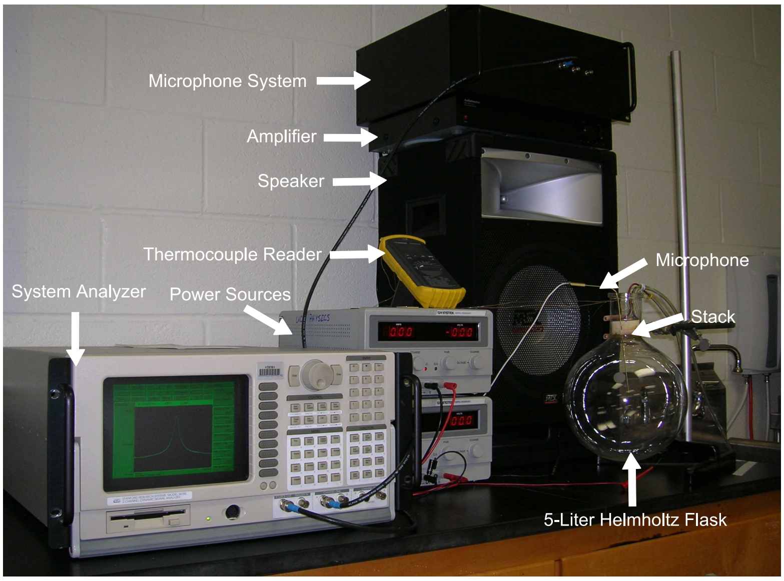


Figure 4: Experimental setup in prior experiment.

1.4 Impedance Measurements

Specific acoustic impedance, z, is defined as the complex ratio of average pressure to velocity at a certain point in an acoustical or mechanical system. [4]

$$z = \frac{P}{u} \tag{13}$$

The specific acoustic impedance is composed of real and imaginary parts:

$$z = r + ix \tag{14}$$

r is referred to as the specific acoustic resistance and x is the specific acoustic reactance. [1]

In order to measure the impedance at a point along the neck of a thermoacoustic engine constructed from a Helmholtz resonator, one would need to take pressure measurements at two locations along the Helmholtz flask's neck and use these measurements in Euler's equation to determine the acoustic velocity, u_{AC}. Inserting the average of these two pressure measurements and the acoustic velocity into Equation 13 yields:

$$z = \frac{P_{AVG}}{u_{AC}} \tag{15}$$

This expression gives the specific acoustic impedance at a specific location between the two microphones.

In order to calculate the impedance at the opening of the flask's neck, one needs to invoke the Impedance Translation Theorem. For a duct with air driven by a source at one end, the equations for the complex pressure and velocity can be written as:

$$\begin{pmatrix} p/\rho_0 c \\ v_x \end{pmatrix} = (ae^{ikx} \pm be^{-ikx})e^{i\omega t} \tag{16}$$

In Equation 16, a and b are constants that are dependent upon the boundary conditions and k is the wave number. [5] Setting $x = 0$ to be the point equidistant from the two microphones, as seen in Figure 5, yields:

$$p(0) = p_{AVG} = \rho_0 c(a+b)e^{i\omega t} \tag{17}$$

and

$$v_x(0) = u_{AC} = (a-b)e^{i\omega t} \tag{18}$$

Inserting Equations 17 and 18 into Equation 15 for the impedance at the position between the two microphones gives:

$$z = \rho_0 c \frac{a+b}{a-b} \tag{19}$$

Letting L be the distance from $x = 0$ to the opening of the neck, as shown in Figure 5, we can define the impedance at the opening to be:

$$Z = \frac{p(L)}{v_x(L)}$$

$$Z = \rho_0 c \frac{ae^{ikL} + be^{-ikL}}{ae^{ikL} - be^{-ikL}} \tag{20}$$

Inserting the exponential definitions for sine and cosine[3] into Equation 20 and simplifying yields:

$$\frac{Z}{\rho_0 c} = \frac{z\cos(kL) + i\rho_0 c\sin(kL)}{\rho_0 c\cos(kL) + iz\sin(kL)} \tag{21}$$

This equation gives the impedance at the opening of the thermacoustic engine's neck.

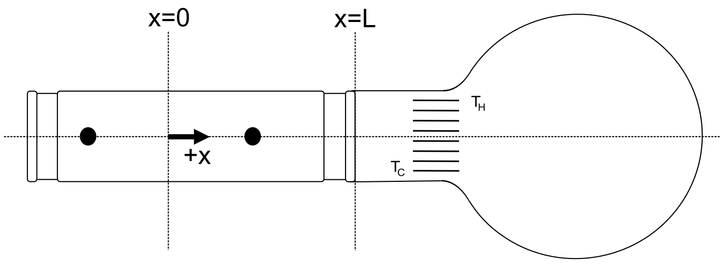


Figure 5: For the experimental setup, x=0 is defined to be the point equidistant from the two microphones, and x=L is at the opening of the flask's neck.

1.5 Reflected and Transmitted Sound Waves.

In order to take measurements of the impedance at the opening of the engine's neck, a closed system must be created by attaching a speaker to the engine with a length of pipe, which houses microphones to measure pressures at two different locations. A schematic of this setup can be seen in Figure 6.

The air within this system is driven by the speaker, which sends an incident sound wave along the length of the pipe. When this incident wave encounters the stack,

[3] $\sin\theta = \frac{e^{i\theta} - e^{-i\theta}}{2i}$ and $\cos\theta = \frac{e^{i\theta} + e^{-i\theta}}{2}$

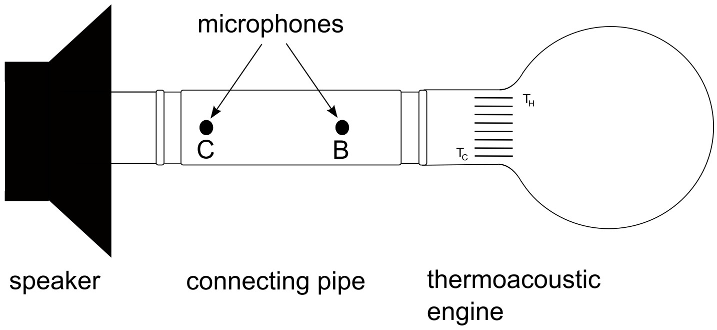


Figure 6: Overview of the closed-system setup used to measure impedance at the opening of the Helmholtz flask.

part of the wave is transmitted and the rest is reflected. The microphones along the length of pipe will therefore measure not only the pressure of the incident wave but a superposition of the pressures of the incident wave and the reflected wave.

$$P_B = P_{\text{incident}} + P_{\text{reflected}} \tag{22}$$

The reflection coefficient, $\mathcal{R}$, describes how much of the incident wave is reflected:

$$\mathcal{R} = \frac{P_{\text{reflected}}}{P_{\text{incident}}} \tag{23}$$

If the incident wave encounters a solid boundary, then $\mathcal{R}$=1, and the incident wave is entirely reflected. However, if no boundary exists, $\mathcal{R}$=0, and the incident wave will continue to propagate through the system. In this setup to measure the impedance, the ceramic stack cannot be treated as a solid boundary because it is porous. However, a portion of the incident wave is reflected upon encountering the stack, while another portion continues to propagate through the system. This reflection and transmission of the incident wave is illustrated in Figure 7.

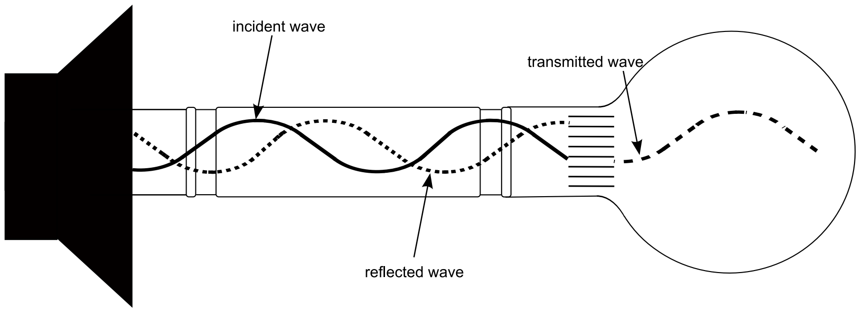
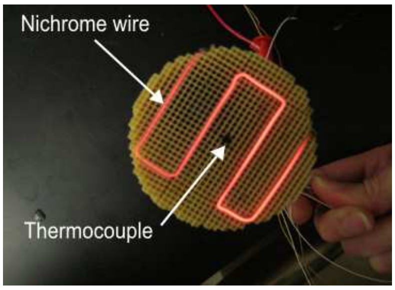


Figure 7: The incident wave reflecting and transmitting upon encountering the stack. Note that the phases of the waves drawn in the figure are for illustrative purposes only.

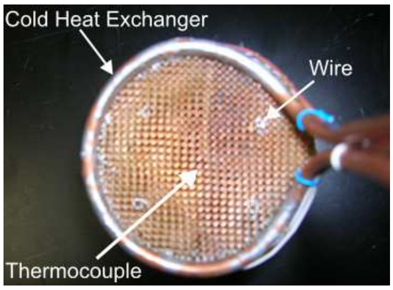
2 Experimental Setup

In this experiment a thermoacoustic engine was constructed from a 5-Liter Helmholtz resonator, that had a neck with a cross-sectional area of 2.03×10^{-3} m^2 and length 8.90×10^{-2} m, by inserting a ceramic stack into the bottom portion of the flask's neck. This 2.54 cm long, 4.75 cm diameter cylindrical stack was constructed from a 324 cells per square inch ceramic substrate. Small grooves were cut onto one face of the stack in preparation of wiring that face with 0.08435 Ω/cm nichrome wire. The nichrome wire served as the hot heat exchanger in this experiment. Both ends of the nichrome wire were connected to approximately fifty-centimeter-long strands of 26-gauge wire. Later in the experiment, both wires would be connected to two parallel GW INSTEK Model GPR-3060D labaratory DC power supplies that would provide the nichrome wire with electrical power, and the current and voltage produced by these power supplies were measured with two Fluke 175 True RMS Multimeters. An Omega Type K thermocouple was fed through the middle pore and adhered inside the pore just at the opening on the face of the stack with the nichrome wire. This thermocouple would measure the temperature on this face of the stack. The hot heat exchanger can be viewed in Figure 8(a).

On the other face of the stack, a copper cold heat exchanger was adhered using



(a) Hot heat exchanger.



(b) Cold heat exchanger.

Figure 8: Bottom and top views of the ceramic stack.

four strands of 30-gauge wire. The cold heat exchanger consisted of a circular portion of copper mesh approximately the diameter of the stack. A 0.3175 cm outer diameter copper tube lined the outer rim of the mesh. During the course of the experiment, 0°C water, provided by a Thermo Fisher Scientific's NESLAB RTE 7 Refrigerated Bath, would be pumped through the tube in order to achieve large temperature differences across the stack. Beneath the cold heat exchanger another thermocouple was placed just within the opening of the middle pore in order to measure the temperature on this face of the stack. The cold heat exchanger can be viewed in Figure 8(b).

The thermoacoustic engine was then attached to an eight inch long, two inch inner diameter PTFE plastic pipe with the use of a pipe connector. Two half-inch diameter holes were drilled four inches apart in the pipe so that two Endevco Model 8510B-1 microphones, that had a flat-frequency response, could take pressure measurements at two different locations in the pipe. Detailed measurements of the pipe can be viewed in Figure 9.

After connecting the thermoacoustic engine to the pipe, the pipe was then attached to a twelve-inch-diameter audio speaker, which would serve to drive the air within the pipe and thermoacoustic engine. The speaker was connected to a Stanford Research Systems SR785 signal analyzer, which controlled the frequency and amplitude of the sine wave created by the speaker. During the entirety of the experiment, the

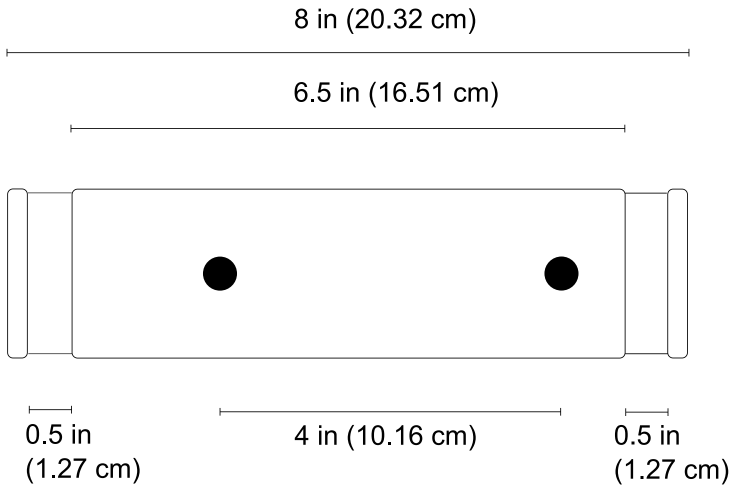


Figure 9: Pipe Connecting Audio Speaker to Thermoacoustic Engine.

amplitude of the signal produced by the signal generator was set to 1.5 Volts, and the thermoacoustic engine was swept through a range of frequencies, 90-150 Hertz. Figure 10 displays the entire experimental setup.

Before water or power were provided to the ceramic stack, the thermoacoustic engine was swept from 90 to 150 Hertz. The signal analyzer recorded the pressure of both microphones as a function of frequency. This procedure was repeated two more times, and the measurements recorded for each microphone were averaged separately.

Afterwards, 0°C water from the refrigerated bath was circulated through the cold heat exchanger, and the temperatures at both faces of the ceramic stack were given time to equilibriate. As before, the thermoacoustic engine was swept from 90 to 150 Hertz, and the procedure was repeated for a total of three trials. The measurements from the three trials for each microphone were then averaged separately.

While the 0°C water circulated through the cold heat exchanger, the power sources were set to provide 0.4 Watts of power to the hot heat exchanger. The temperatures on side of the ceramic stack were given time to equilibriate, and a temperature difference of 9°C was produced. The thermoacoustic engine was then swept over the 90-150 Hertz range. A total of three experimental trials were performed at this temperature

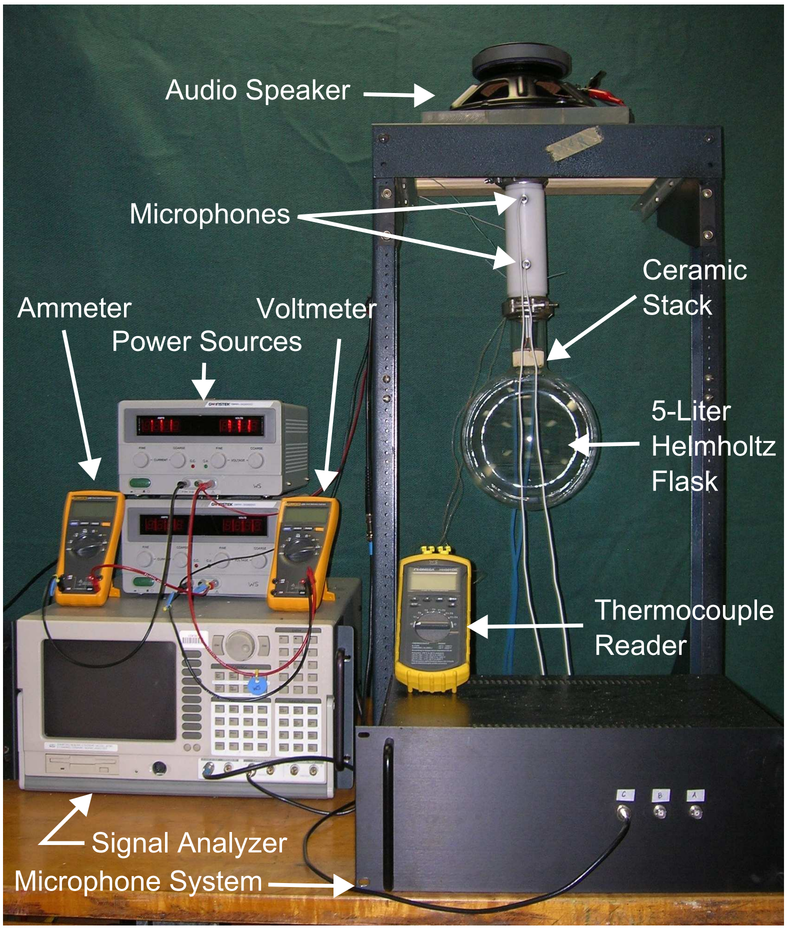


Figure 10: Experimental Setup.

difference. This procedure was repeated for eight increasing temperature differences across the ceramic stack.

3 Results and Analysis

Figure 11 displays the pressure recorded by microphone B (refer to Figure 7) at various temperature differences across the stack as the engine was swept over a range of frequencies. At $\Delta T=0°$, no water from the refrigerated bath or power were provided to the stack. At $\Delta T=5°$, water was circulated through the cold heat exchanger, but power was not provided to the hot heat exchanger. By allowing water to flow through

ΔT Across the Stack (°C)	T_H (°C)	T_C (°C)	Power (Watts)
0	22	22	0
5	13	8	0
9	19	10	0.396
21	38	17	1.60
40	67	27	1.48
65	107	42	6.66
91	148	57	10.6
116	189	73	15.8
136	223	87	21.9
150	246	96	27.7
176	283	107	36.1

Table 1: Thermocouples on either side of the ceramic stack measured the temperatures and ΔT was calculated from their difference. T_H corresponds to the temperature on the side of the stack with the hot heat exchanger whereas T_C corresponds to the temperature on the side with the cold heat exchanger. The column with the power values corresponds to the amount of electrical power provided to the nichrome wire in order to achieve the different temperature differences.

the cold heat exchanger, the density and speed of sound of the air decreased from their values at ΔT=0°, causing the pressure recorded by microphone B to shift upwards. However, one can see that the overall range for the acoustic pressure amplitudes is decreasing as the temperature difference across the stack increases from ΔT=5°. Referring to Equation 22, the incident pressure does not change because the speaker is driving the air within the system at a set amplitude from the signal analyzer; therefore, the reflected wave must decrease in order to correspond with this decrease in total acoustic pressure. Essentially, the reflecion coefficient, $\mathcal{R}$, for this system is decreasing as the temperature difference increases. Therefore, one can assume that by increasing the temperature difference of the stack, the resistance in the system is being eliminated.

A Mathematica notebook, which can be viewed in Appendix C, has been created to calculate the impedance at the neck of the thermoacoustic engine. At this time, the impedance at the opening of the flask has not been calculated because measurements

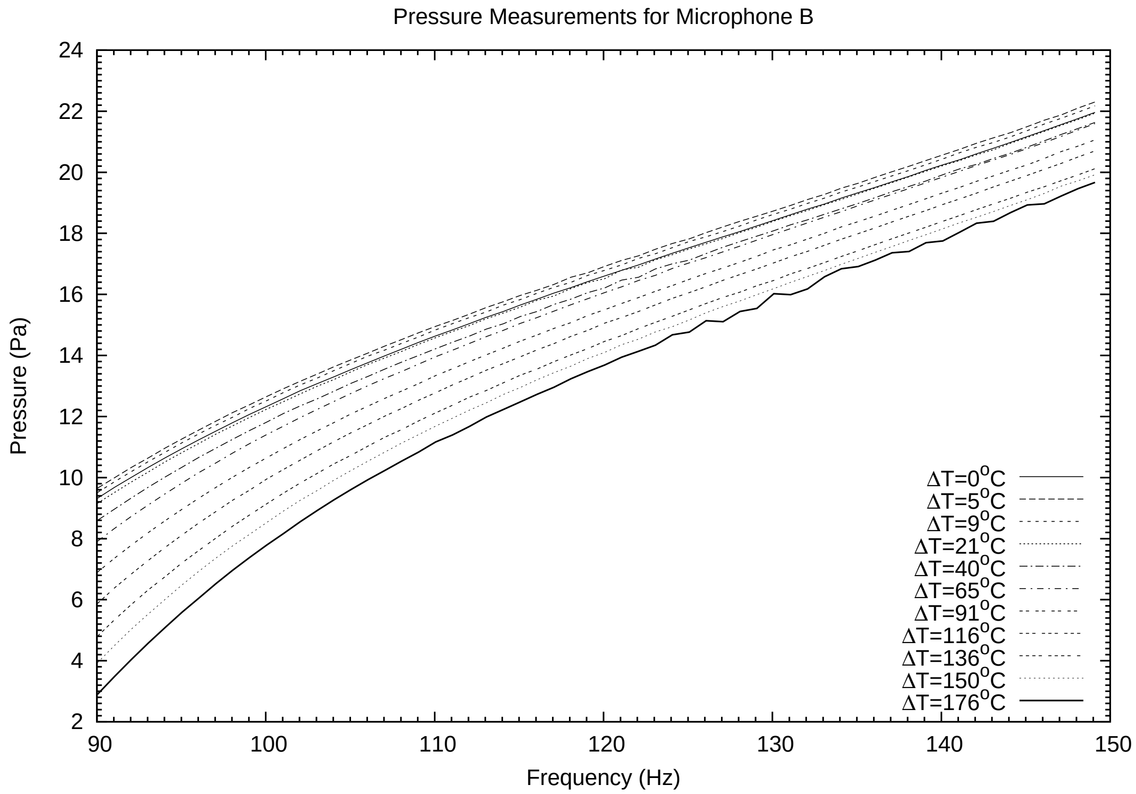


Figure 11: Pressure measured by microphone B as a function of frequency. Pressure measurements were made for eleven increasing temperature differences across the stack.

of the phase for the recorded pressure are still being collected.

4 Conclusion

A thermoacoustic engine is a heat engine that uses heat energy to perform work in the form of sound. A prior experiment showed that this particular engine reached onset at a temperature difference of 207°C across its stack. The measurements taken during this prior experiment showed that the quality factor for this system increased as the temperature differences increased towards the onset temperature difference. This increase in the quality factor corresponds to a decrease in the system's resistance.

However, the measurements made in the prior experiment primarily were directly related to the specific acoustic resistance, or the real part of the impedance, and were not easily relatable to the specific acoustic reactance. In order to explore both the real and imaginary parts of the impedance, a closed system was created such that two different microphone measurements could collect two different pressure measurements simultaneously as the engine was swept over a range of frequencies. By calculating the average pressure and the acoustic velocity at a point between these two microphones, it is possible to determine the impedance at the opening of the engine's neck by invoking the Impedance Translation Theorem. During this time a Mathematica notebook has been created to calculate the impedance at the opening of the engine's neck; however; measurements for the pressure's phase within the system are currently being collected.

At temperature differences below the onset temperature, the incident sound wave is partially reflected upon encountering the ceramic stack, while being partially transmitted. Pressure measurements from microphone B show that as the temperature difference across the stack increases, the pressure is decreasing. Because this recorded pressure is a superposition of the incident and reflected wave, the reflected pressure must be decreasing as the temperature difference increases. Therefore, as the temperature difference increases, the reflection coefficient is decreasing, and one can assume that the resistance in the system is decreasing.

Appendix A: Derivation of Euler's Equation

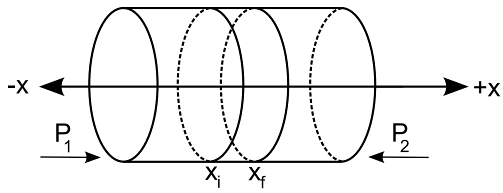


Figure 12: Pipe with pressures P_1 and P_2 at either openings.

Consider a pipe as shown in Figure 12. Assume that the pressure P_2 is greater than the P_1; therefore, the change in pressure, ΔP, is negative. Because pressure is defined as the force per unit area, A, Newton's Second Law can be written in terms of the pressure.

$$\begin{aligned} \Sigma\vec{F} &= m\vec{a} \\ -A\Delta P &= ma \end{aligned} \tag{24}$$

Because acceleration is the first derivative of the velocity, Equation 24 can be written as:

$$-A\Delta P = m\frac{dv}{dt} \tag{25}$$

Considering a pressure change over a small volume in the pipe, $V = A(x_f - x_i) = A\Delta x$, and dividing both sides of Equation 25 by this volume yields:

$$\begin{aligned} -\frac{A\Delta P}{A\Delta x} &= \frac{m}{A\Delta x}\frac{dv}{dt} \\ -\frac{\Delta P}{\Delta x} &= \frac{m}{A\Delta x}\frac{dv}{dt} \end{aligned} \tag{26}$$

The density of the air for ambient conditions, ρ_o, is defined as the mass per unit volume of air:

$$\begin{aligned}\rho_o &= \frac{m}{V} \\ \rho_o &= \frac{m}{A\Delta x}\end{aligned} \tag{27}$$

Inserting this value for the ambient air density into Equation 26 gives:

$$-\frac{\Delta P}{\Delta x} = \rho_o \frac{dv}{dt} \tag{28}$$

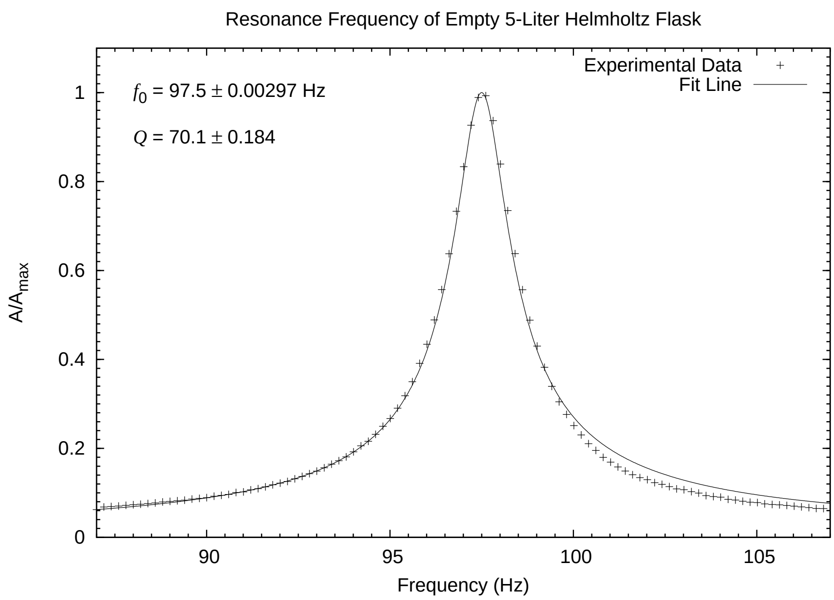
Due to the sinusoidal nature of the pressure and velocity, the pressure and velocity are represented by $P = P_o(x)e^{i\omega_o t}$ and $v = u_{AC}(x)e^{i\omega_o t}$, respectively. Inserting this value for pressure and the derivative of the velocity with respect to time into Equation 28 yields:

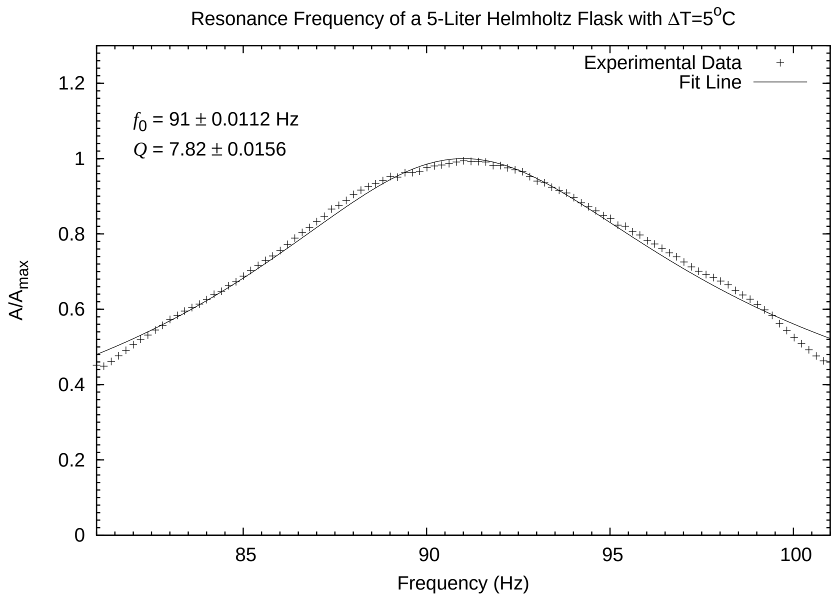
$$\begin{aligned}\frac{-\Delta P_o(x)e^{i\omega_0 t}}{\Delta x} &= (i\omega_o\rho_o)(u_{AC}(x)e^{i\omega_0 t}) \\ \frac{-\Delta P_o(x)}{\Delta x} &= (i\omega o\rho_o)(u_{AC}(x)) \\ u_{AC} &= \left|\frac{1}{\rho_o\omega_o}\frac{\Delta P_o}{\Delta x}\right|\end{aligned} \tag{29}$$

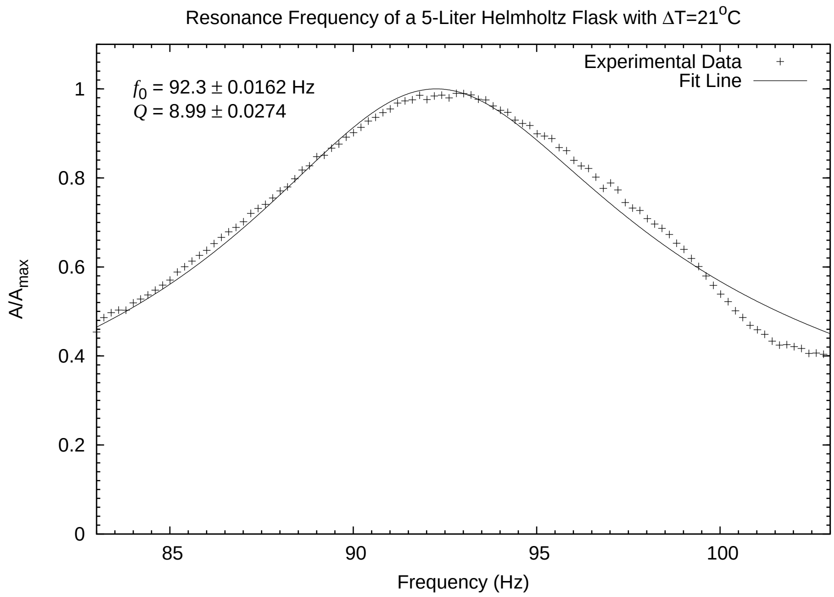
Appendix B: Summary of the Results from a Prior Experiment

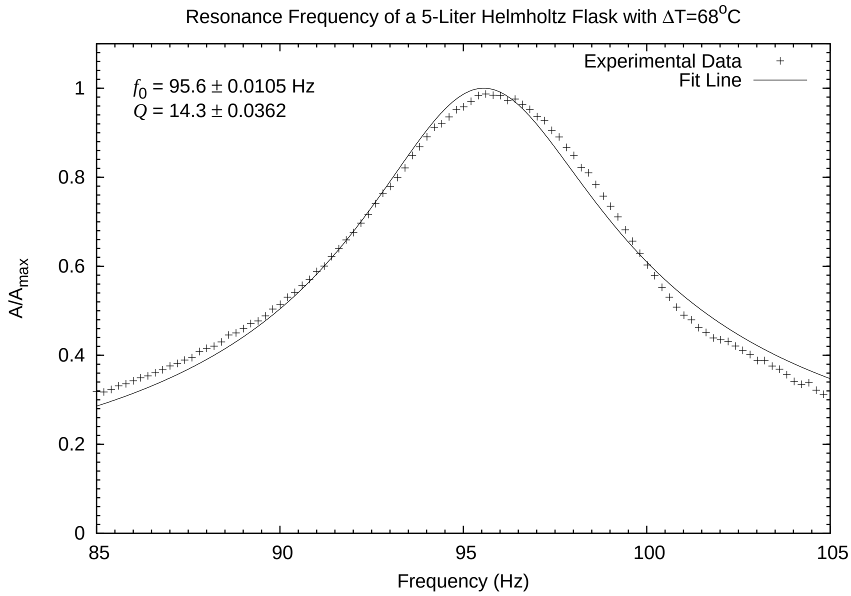
ΔT Across Stack(°C)	Resonance Frequency (Hertz)	Quality Factor
0 (Empty Flask)	97.5 ± 0.00297	70.1 ± 0.184
5	91.0 ± 0.0112	7.82 ± 0.0156
21	92.6 ± 0.0162	8.99 ± 0.0274
68	95.6 ± 0.0105	14.3 ± 0.0362
121	99.7 ± 0.00322	21.6 ± 0.0218
166	104.9 ± 0.00345	35.6 ± 0.0555
196	108.3 ± 0.00330	52.8 ±0.108
203	109.0 ± 0.00375	56.3 ± 0.138

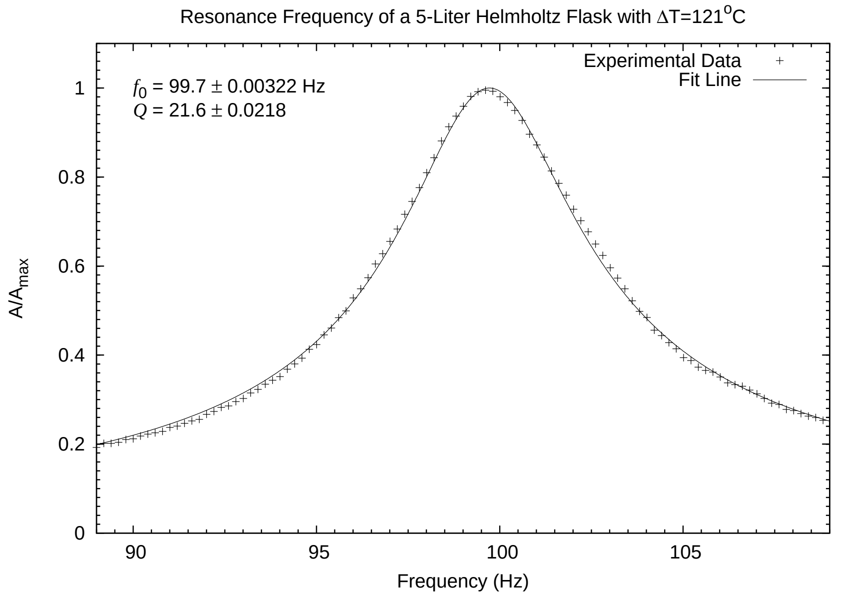
Table 2: For different temperature differences across the stack, the resonance frequency and quality factor for the thermoacoustic engine were determined. As the temperature difference increases, the resonance frequency increases, as well, due to the increase in the speed of sound as the temperature within the flask increases. The quality factor is shown to increase with increasing temperature differences, which is associated with a decrease in the resistance.

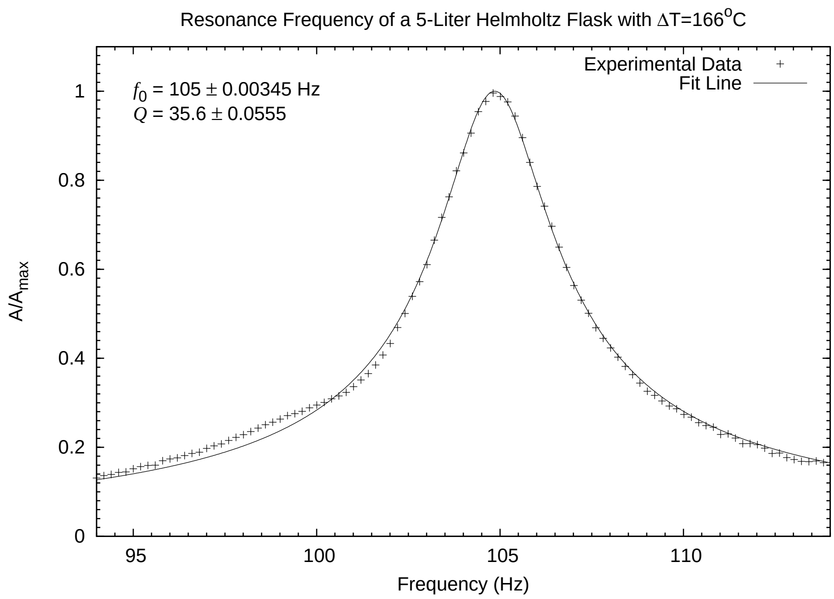



Resonance Frequency of a 5-Liter Helmholtz Flask with ΔT=5°C
Experimental Data
Fit Line
f0 = 91 ± 0.0112 Hz
Q = 7.82 ± 0.0156
A/Amax
Frequency (Hz)
0
0.2
0.4
0.6
0.8
1
1.2
85
90
95
100

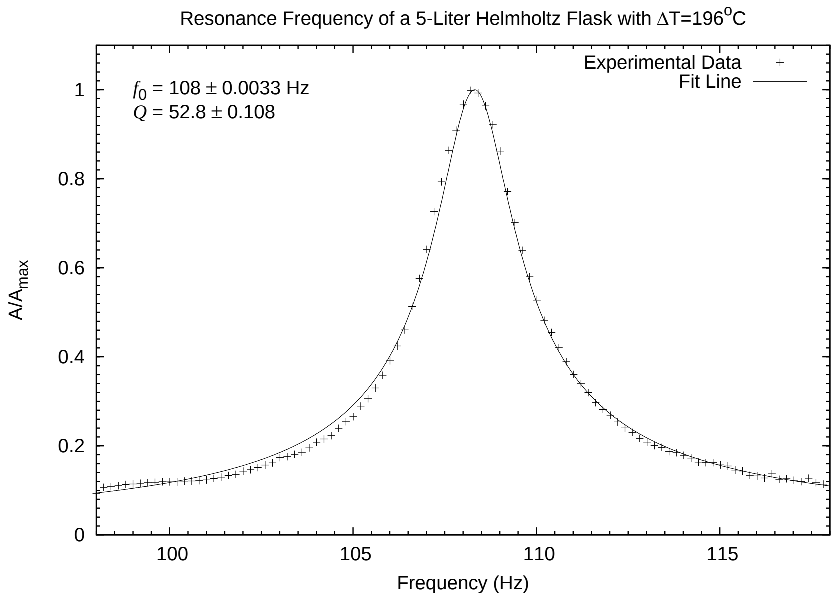

Resonance Frequency of a 5-Liter Helmholtz Flask with ΔT=21°C
Experimental Data
Fit Line
f0 = 92.3 ± 0.0162 Hz
Q = 8.99 ± 0.0274
A/Amax
Frequency (Hz)
0
0.2
0.4
0.6
0.8
1
85
90
95
100


Resonance Frequency of a 5-Liter Helmholtz Flask with ΔT=68°C
Experimental Data
Fit Line
f_0 = 95.6 ± 0.0105 Hz
Q = 14.3 ± 0.0362
A/A_{max}
Frequency (Hz)

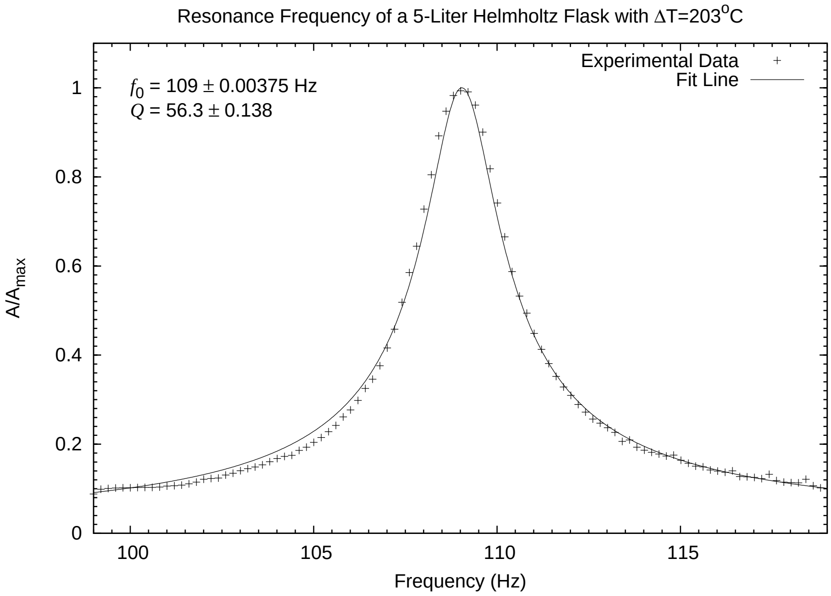

Resonance Frequency of a 5-Liter Helmholtz Flask with ΔT=121°C
Experimental Data
Fit Line
f_0 = 99.7 ± 0.00322 Hz
Q = 21.6 ± 0.0218
A/A_{max}
Frequency (Hz)

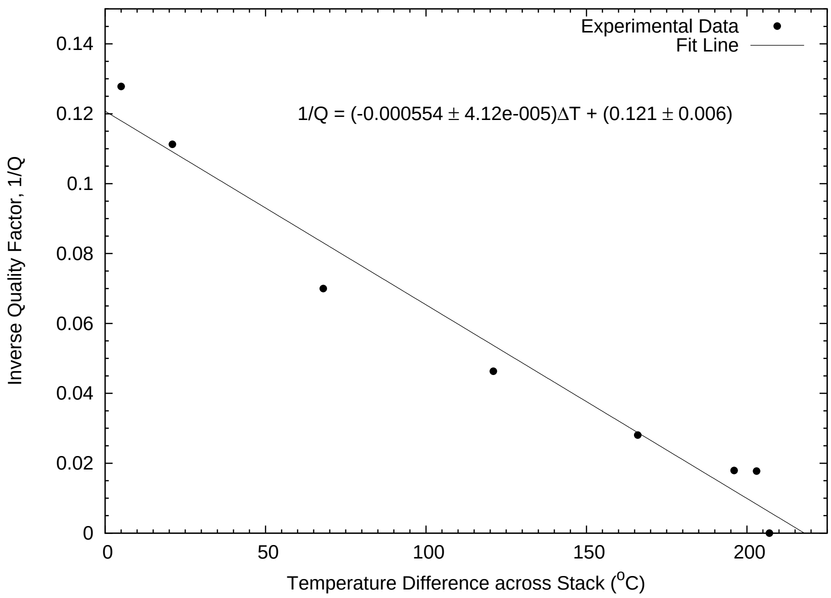


Resonance Frequency of a 5-Liter Helmholtz Flask with ΔT=166°C
Experimental Data
Fit Line
f_0 = 105 ± 0.00345 Hz
Q = 35.6 ± 0.0555
A/A_max
Frequency (Hz)



Resonance Frequency of a 5-Liter Helmholtz Flask with ΔT=196°C
Experimental Data
Fit Line
f_0 = 108 ± 0.0033 Hz
Q = 52.8 ± 0.108
A/A_max
Frequency (Hz)


Resonance Frequency of a 5-Liter Helmholtz Flask with ΔT=203°C
Experimental Data
Fit Line
f_0 = 109 ± 0.00375 Hz
Q = 56.3 ± 0.138
A/A_{max}
Frequency (Hz)


Experimental Data
Fit Line
1/Q = (-0.000554 ± 4.12e-005)ΔT + (0.121 ± 0.006)
Inverse Quality Factor, 1/Q
Temperature Difference across Stack (°C)

Appendix C: Mathematica Notebook

This sample from the Mathematic notebook creates a file that lists the frequency at which the engine is being driven, the real impedance, and the imaginary impedance.

References

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[2] Steven L. Garrett. "Resource Letter: TA-1: Thermoacoustic engines and refrigerators," American Journal of Physics **72**(1), 11-17 (2004).

[3] G.W. Swift. *Thermoacoustics– A Unifying Perspective for Some Engines and Refrigerators*, Acoustical Society of America, Melville, NY, 2002.

[4] Beranek, Leo L. *Acoustics*, Acoustical Society of America, Woodbury, NY, 1996.

[5] Pierce, Allan D. *Acoustics: An Introduction to Its Physical Properties and Applications*, 2nd Edition, Acoustical Society of America, Melville, NY, 1989.