Behavior of a Helmholtz Resonator Driven at High Amplitudes

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Abstract

The acoustic behavior of a Helmholtz resonator was studied when driven by a compressed air source. The resonator consists of a 55 gallon drum with 4" diameter necks of different lengths. Compressed air from a 0-15psi regulator is introduced into the resonator using an electronic valve controlled by a signal generator. A pressure sensor was used to study the acoustic behavior of the resonator as it was driven over the resonance frequency with the compressed air source. By closely examining the resonant peaks, the quality factor of the system could be determined for different drive pressures. The measured resonance frequencies are compared to two theoretical models. The resonator's quality factor (energy stored in resonance/energy lost per cycle) is shown to decrease with drive amplitude indicating increased losses with higher amplitude.

Experimental Setup

Helmholtz Resonator System with 1.0m Neck Installed

The apparatus consists of a 55 gallon drum, with a ½" thick aluminum lid sealed using a bicycle tire inner-tube and c-clamps. The lid has a 4" diameter hole cut in the center, around which are mounting points for a 4" diameter neck. Three different length tubes of 1.0m, 0.25m and 0.5m were available to alter the Helmholtz neck length as needed. Two additional ports are present on the aluminum lid for the pressure sensor and the compressed air inlet. The compressed air supply is connected directly to the Lewis Science Center's built-in compressed air system and is controlled by a 0-15psi regulator and a fast switching electronic valve.

The pressure sensor is constructed of a Freescale Semiconductor integrated pressure sensor. This sensor has a maximum pressure rating of 10kPa and a response time of 1.0ms, which is sufficient for our system. The pressure sensor also contains additional circuitry for filtering, an L.E.D. indicator, signal output and input ports, and a DC power supply port. The sensor was calibrated using a water manometer.

The electronic valve is from SMC. It has a response time of 10ms and a maximum pressure rating of 0.5MPa. This allows for a maximum drive frequency of 50Hz, which is well above that required of about 15Hz.

The neck length of the resonator could be varied by connecting the three available lengths together or using them individually. Each neck length exhibited different resonant frequencies and acoustic behavior.

Determining Resonant Frequency

Theoretically determining the resonant frequency of the system can be accomplished most easily using the usual equation of resonance for a Helmholtz resonator:

$$f = \frac{c}{2\pi}\sqrt{\frac{S}{L'V}} \qquad (1)$$

Where, *c* is the speed of sound, *S* is the cross sectional area of the neck, *V* is the volume of the large cavity and *L'* is the effective length of the neck. For an unflanged, open end, the effective length, *L'* is:

$$L' = L + 1.5a \qquad (2)$$

Where *L* is the actual length of the neck and *a* is the radius of the neck.

The resonant frequency can also be calculated by examining how pressure waves would move throughout the system. By setting up two separate wave equations, one for the pressure wave in each section of the resonator, and exploiting boundary conditions within the system, a relationship can be developed allowing one to calculate the resonant frequency of the system. The resulting equation is:

$$Tan(kL')Tan(kL_{Vol}) - \frac{(R_{Neck})^2}{(R_{Vol})^2} = 0 \qquad (3)$$

Where *L'* is the effective length of the neck, L_{Vol} is the length of the drum, R_{Neck} is the radius of the neck, R_{Vol} is the radius of the drum and *k* is the wave number. By plotting the value of (3) against the wave number, a graph can be produced in which the points where the function crosses the x-axis refer to resonant modes of the system. The resonant frequency can be determined from the value of *k* at this point by the equation:

$$f = \frac{ck}{2\pi} \qquad (4)$$

Where *c* is the speed of sound and *k* is the value at which the function given above is zero.

The Quality Factor

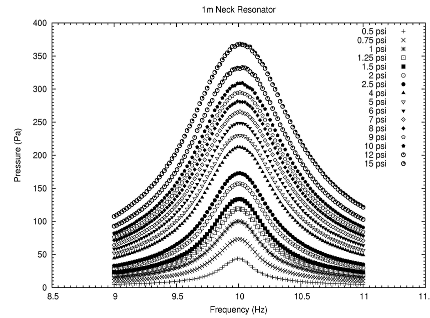
The quality factor of the system's resonance is related to the energy lost in the system. This value is determined by finding the two frequencies at which the value of the pressure is:

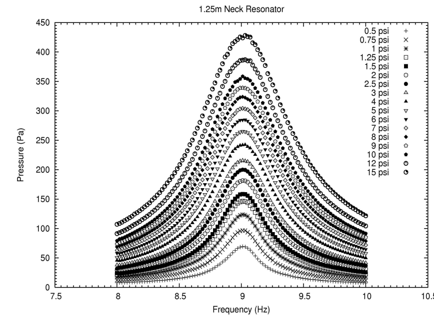
$$\frac{P_{\max}}{\sqrt{2}} \qquad (5)$$

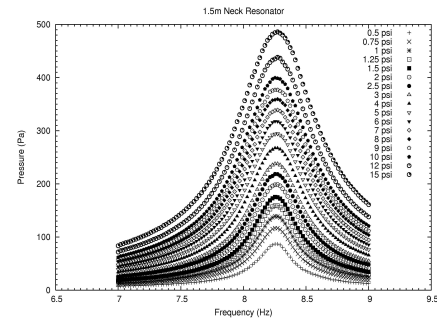
The quality factor is then determined from the equation:

$$Q = \frac{f_0}{|f_2 - f_1|} \qquad (6)$$

Raw Data



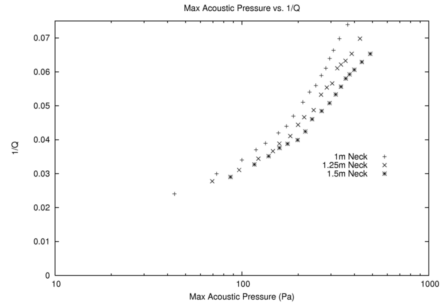




Analyzed Data

Neck Length	Helmholtz Equation Prediction	Wave Equation Prediction	Measured Value
1.0m	10.33Hz	9.98Hz	10.00±.01Hz
1.25m	9.32Hz	9.00Hz	9.01±.01Hz
1.5m	8.56Hz	8.25Hz	8.26±.01Hz

The values above were calculated using the same values for the speed of sound and the various parameters for the size of the Helmholtz resonator. As the chart shows, the wave equation method yields much more accurate values for the resonant frequencies of the system compared to the Helmholtz resonator equation. This is due to the fact that the wave equation prediction more realistically models the system as a superposition of propagating waves, while the Helmholtz equation crudely treats the system as a mass on a spring, where the air inside the neck is a mass, and the air inside the drum is a spring.



The figure above shows the inverse quality factor (1/Q) vs the maximum acoustic pressure amplitude for each drive amplitude and each neck length. The inverse quality factor is a measure of the losses in the system relative to the energy stored in the system. The data shows increased losses with increased amplitude. Understanding this behavior is the subject of future work.

Future Work

The next step of this project will be to theoretically predict the value of Q as a function of the maximum pressure of the system. This can then be compared to the above 1/Q vs. Pmax graph to see how well it predicts the behavior of the system at all drive almplitudes and all three neck lengths.

Acknowledgements

UCA Physics Dept.